EE 230
Lecture 12

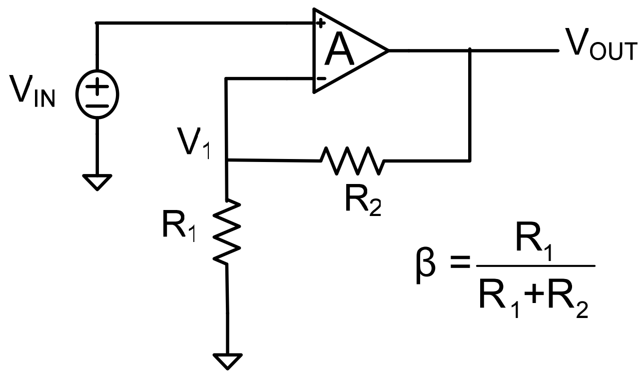
Basic Feedback Configurations

Generalized Feedback Schemes

- Integrators
- Differentiators
- First-order active filters
- Second-order active filters

Review from Last Time

Input and Output Impedances with Feedback

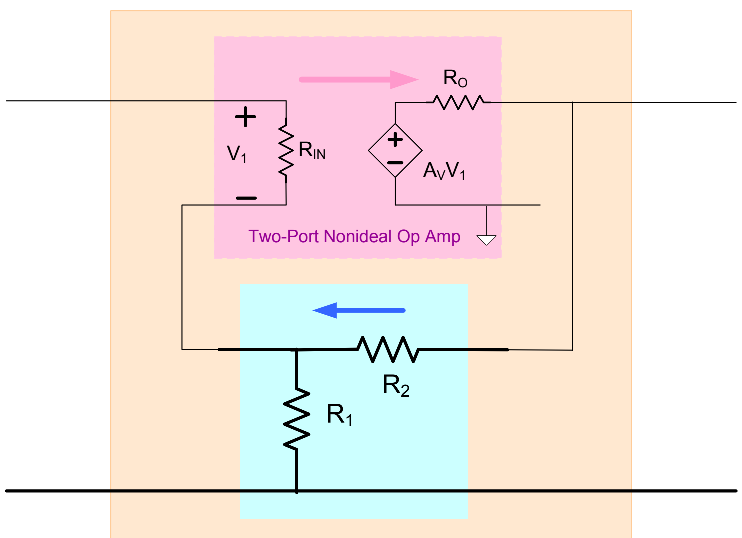


$$\beta = \frac{R_1}{R_1+R_2}$$

R_{INF}=?

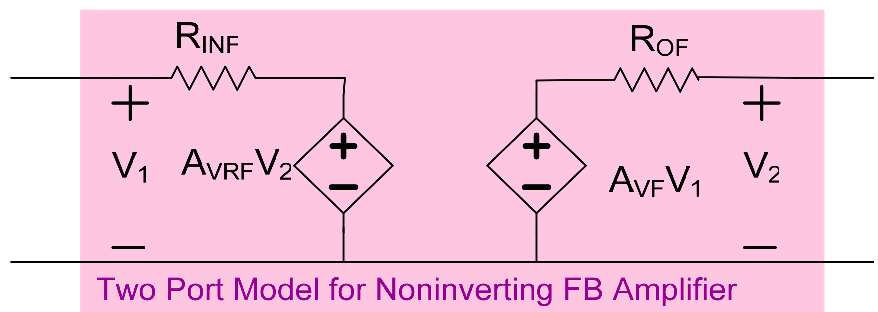
R_{OF}=?

Exact analysis : Consider amplifier as a two-port and use open/short analysis method



Will find R_{INF}, R_{OF}, A_V almost identical to previous calculations

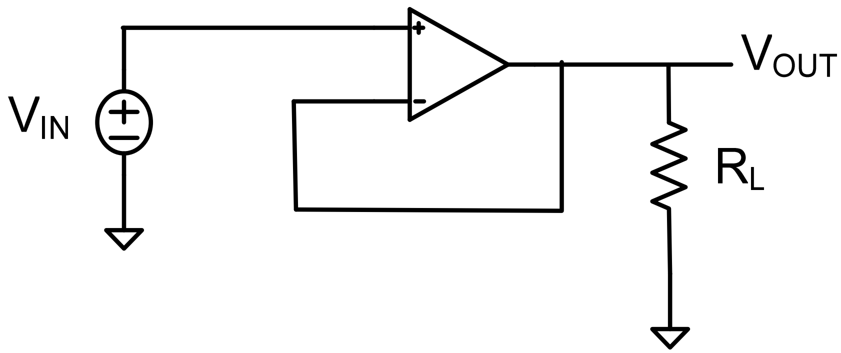
Will see a small A_{VR} present but it plays almost no role since R_{INF} is so large (effectively unilateral)



$$A_{VF} \simeq \frac{1}{\beta} \qquad R_{0F} \simeq \frac{R_0}{1+\beta A_V} \qquad R_{INF} = R_{IN}\left(1+A_V\beta\right) \qquad A_{VRF} \simeq \beta$$

Review from Last Time

Buffer Amplifier



$$A_V = \frac{V_{OUT}}{V_{IN}} = 1$$

$$R_{IN} = \infty$$

$$R_{OUT} = 0$$

One of the most widely used Op Amp circuits

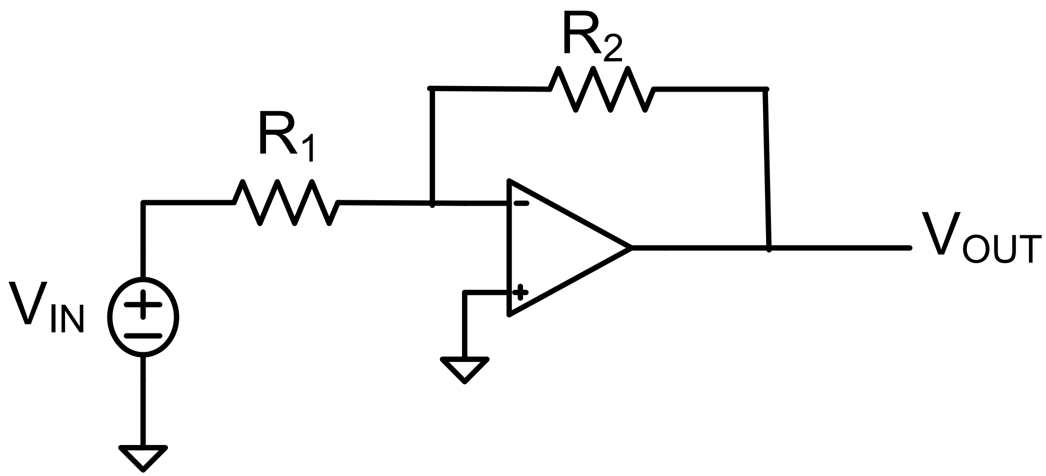
Provides a signal to a load that is not affected by a source impedance

This provides for decoupling between stages in many circuits

Special case of basic noninverting amplifier with $R_1=\infty$ and $R_2=0$

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_2}{R_1}$$

Basic Inverting Amplifier



$$\frac{V_{IN}}{R_1} + \frac{V_{OUT}}{R_2} = 0$$

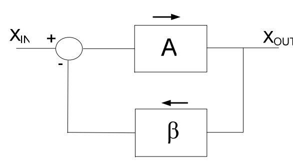
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}$$

$$R_{OUT} = 0$$

$$R_{IN} = R_1$$

Input impedance of R_1 is unacceptable in many (but not all) applications

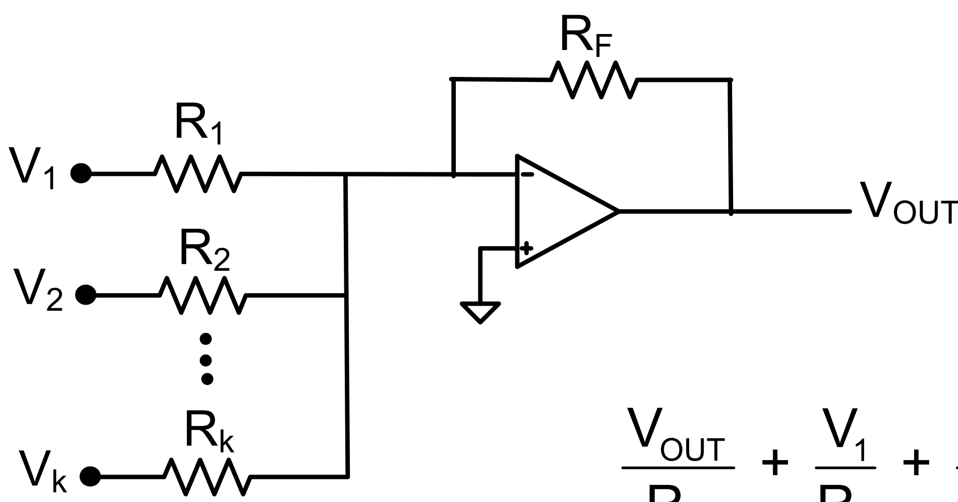
This is not a <u>voltage</u> feedback amplifier (it is a feedback amplifier) of the type (note R_{IN} is not high!)



Feedback concepts could be used to analyze this circuit but lots of detail required

Review from Last Time

Summing Amplifier



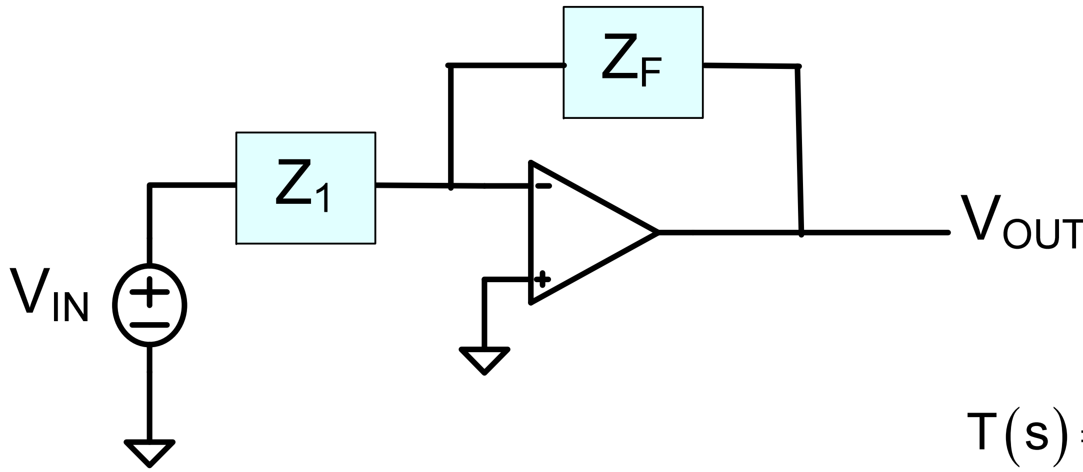
$$\frac{V_{OUT}}{R_F} + \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_k}{R_k} = 0$$

$$V_{OUT} = -\frac{R_F}{R_1}V_1 - \frac{R_F}{R_2}V_2 - \ldots - \frac{R_F}{R_k}V_k$$

- Output is a weighted sum of the input voltages
- Any number of inputs can be used
- Gains from all inputs can be adjusted together with R_F
- Gain for input V_i can be adjusted independently with R_i for $1 \leq I \leq k$
- All weights are negative
- Input impedance on each input is R_i

Generalized Inverting Amplifier

s-domain representation



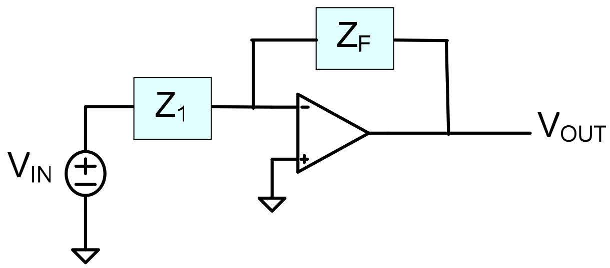
$$T(s) = -\frac{Z_F(s)}{Z_1(s)}$$

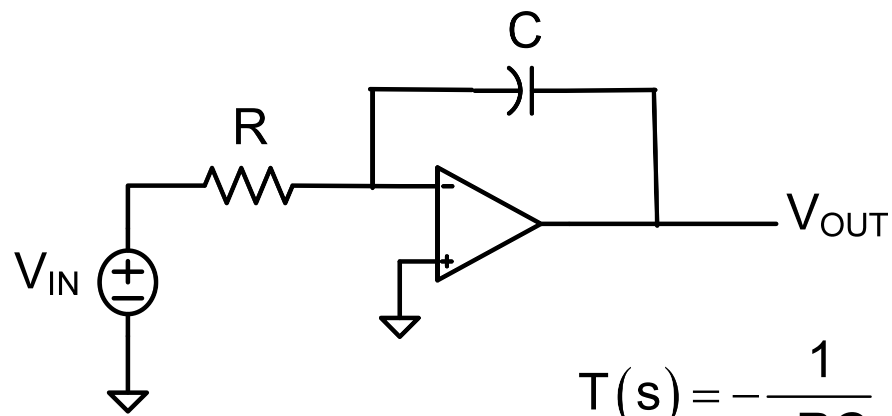
Z_1 and Z_F can be any s-domain circuits

If Z_1=R, Z_F=1/sC, obtain $$T(s) = -\frac{1}{sRC}$$

What is this circuit?

Generalized Inverting Amplifier





$$T(s) = -\frac{1}{sRC}$$

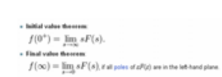
What is this circuit?

Consider the differential equation $$y = K\int_0^t x(\tau)\,d\tau$$

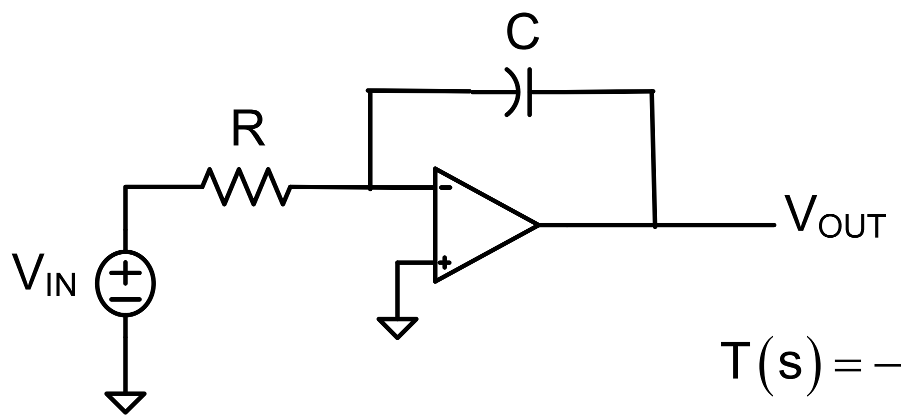
Taking the Laplace Transform, obtain $$Y(s) = K\frac{X(s)}{s}$$ $$T(s) = \frac{Y(s)}{X(s)} = \frac{K}{s}$$

K is the frequency where $|T(j\omega)|=1$ and is termed the Integrator Unity Gain Frequency

Thus, this circuit is an <u>inverting</u> integrator with a unity gain frequency of $K = (RC)^{-1}$



Inverting Integrator



$$T(s) = -\frac{1}{sRC}$$

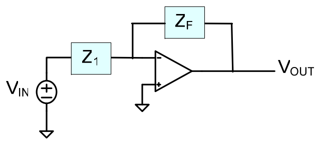
$$T(j\omega) = -\frac{1}{j\omega RC}$$

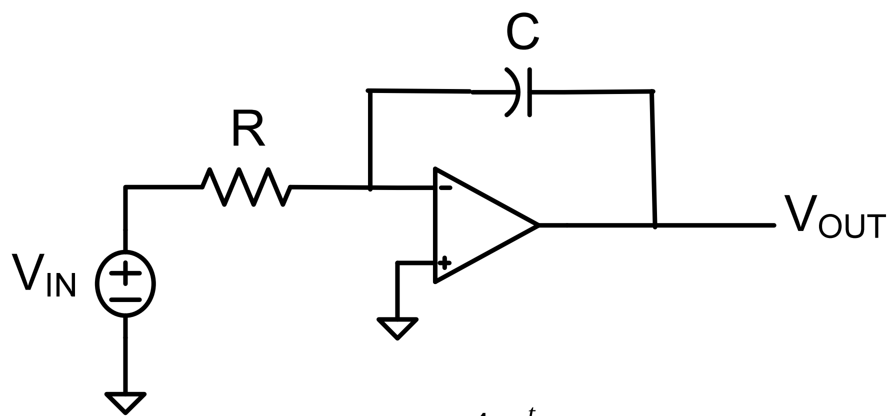
$$\left|T(j\omega)\right| = \frac{1}{\omega RC}$$

$$\angle T(j\omega) = 90^o$$

Unity gain frequency is $\omega_0 = \dfrac{1}{RC}$

Inverting Integrator





$$T(s) = -\frac{1}{sRC}$$

$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

Integrators are widely used !

$R_{IN}=R$ $\qquad$ $R_{OUT}=0$

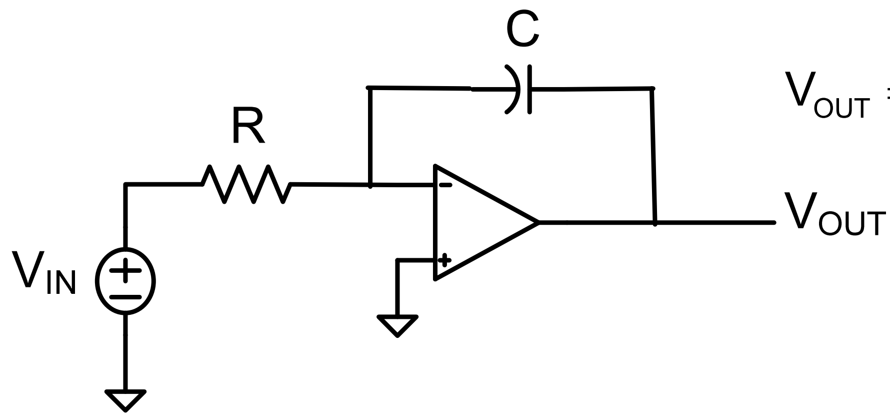
The integrator function itself is ill-conditioned and integrators are seldom used open-loop

The ideal integrator has a pole at s=0 which is not in the LHP

If the input has any dc component present, since superposition applies, the output would diverge to ±∞ as time increases

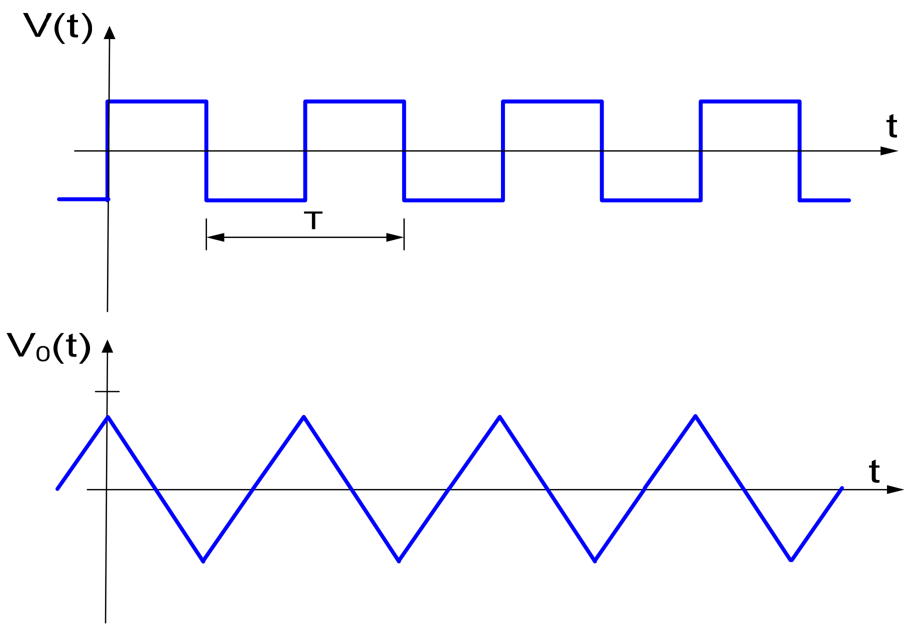
The offset voltage (discussed later) will also cause an integrator output to diverge

Inverting Integrator



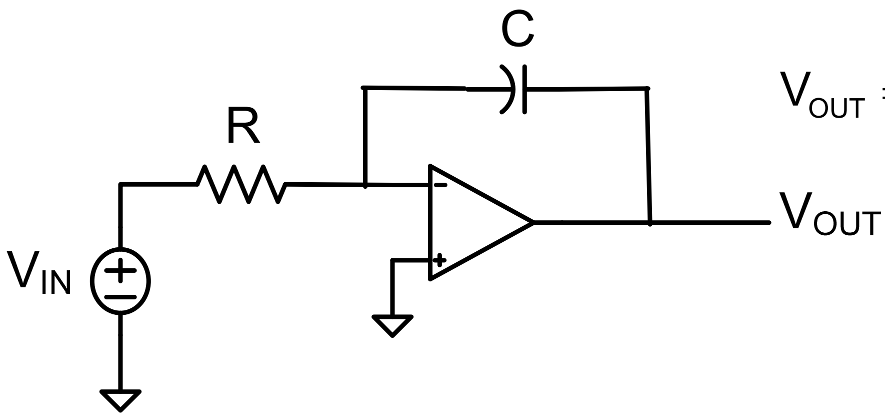
$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

What is the output of an ideal integrator if the input is an ideal square wave?



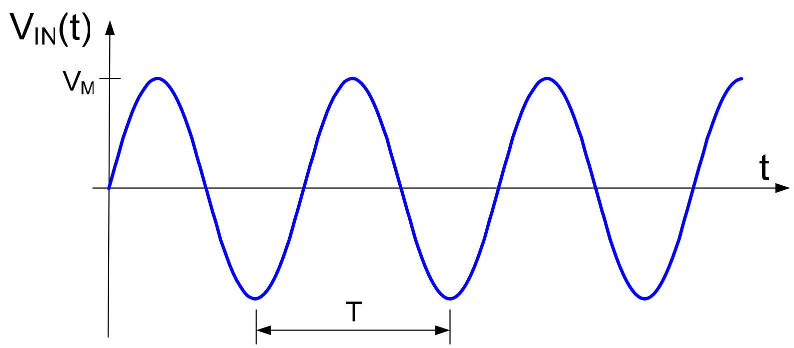
Amplitude of output dependent upon RC product

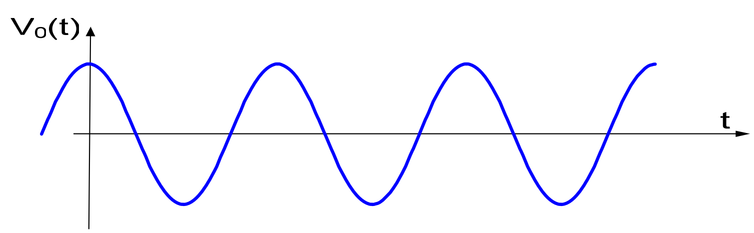
Inverting Integrator



$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

What is the output of an ideal integrator if the input is an ideal sine wave?

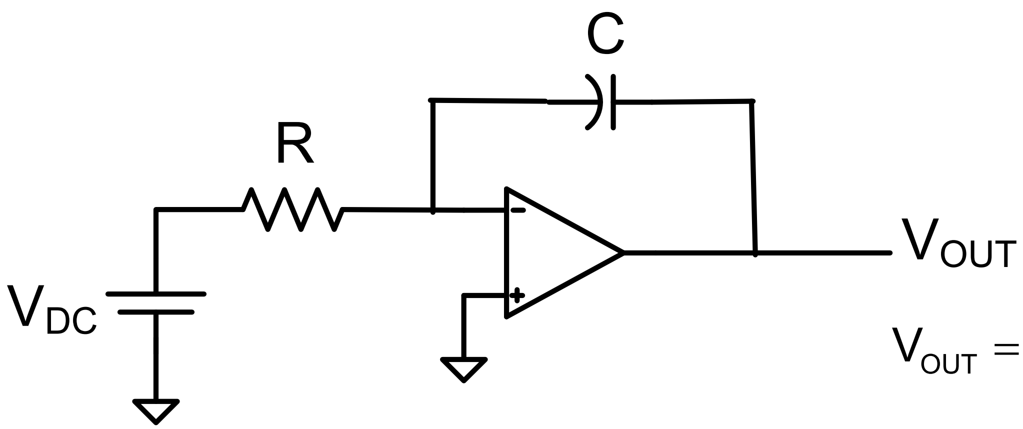




Amplitude of output dependent upon RC product

Inverting Integrator

Ill-conditioned nature of open-loop integrator



$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

If $V_{IN}(0) = 0$,

$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{DC}\,d\tau$$

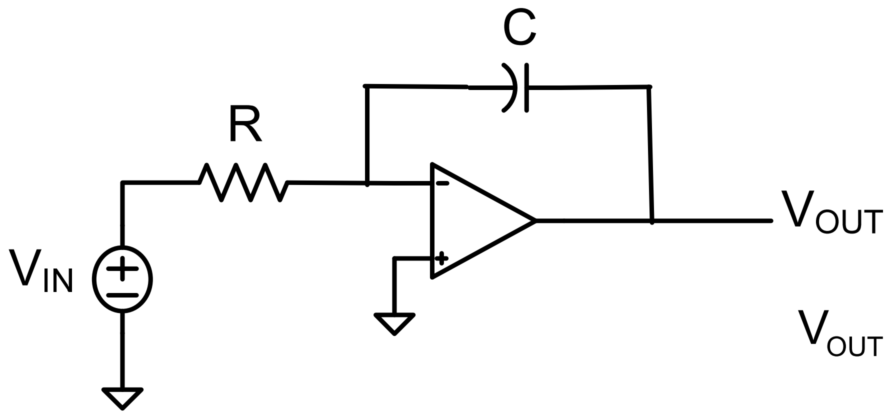
$$V_{OUT} = -\frac{V_{DC}}{RC}\int_0^t 1\,d\tau$$

$$V_{OUT} = -\frac{V_{DC}}{RC}\left(\tau\Big|_0^t\right) = -\frac{V_{DC}}{RC}t$$

For any values of V_{DC}, R, and C, the output will diverge to ± ∞

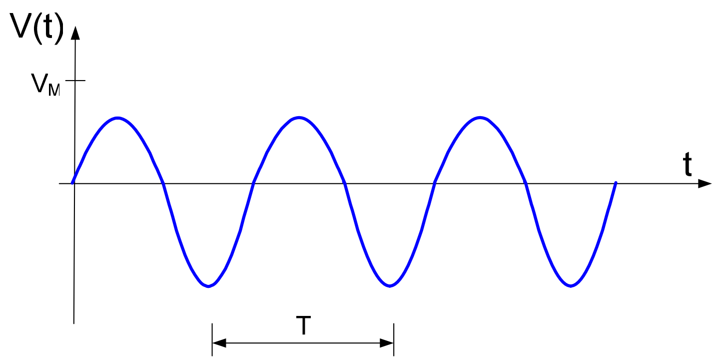
Inverting Integrator

Ill-conditioned nature of open-loop integrator



$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

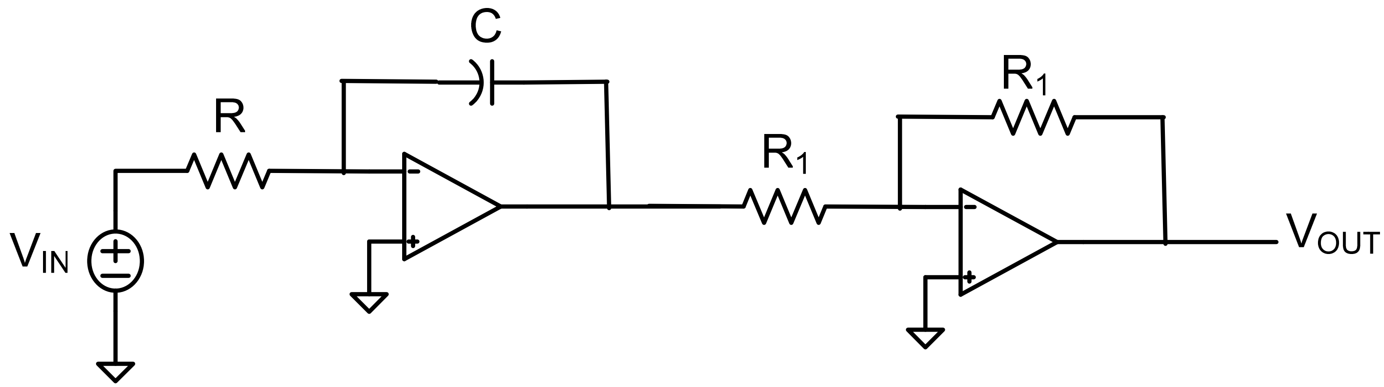
Any periodic input that in which the average value is not EXACTLY 0 will have a dc component



$$V_{IN}(t) = A_0 + \sum_{k=1}^{\infty} \sin(k\omega t + \theta_k)$$

This dc input will cause the output to diverge!

Noninverting Integrator



$$V_{OUT} = \frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

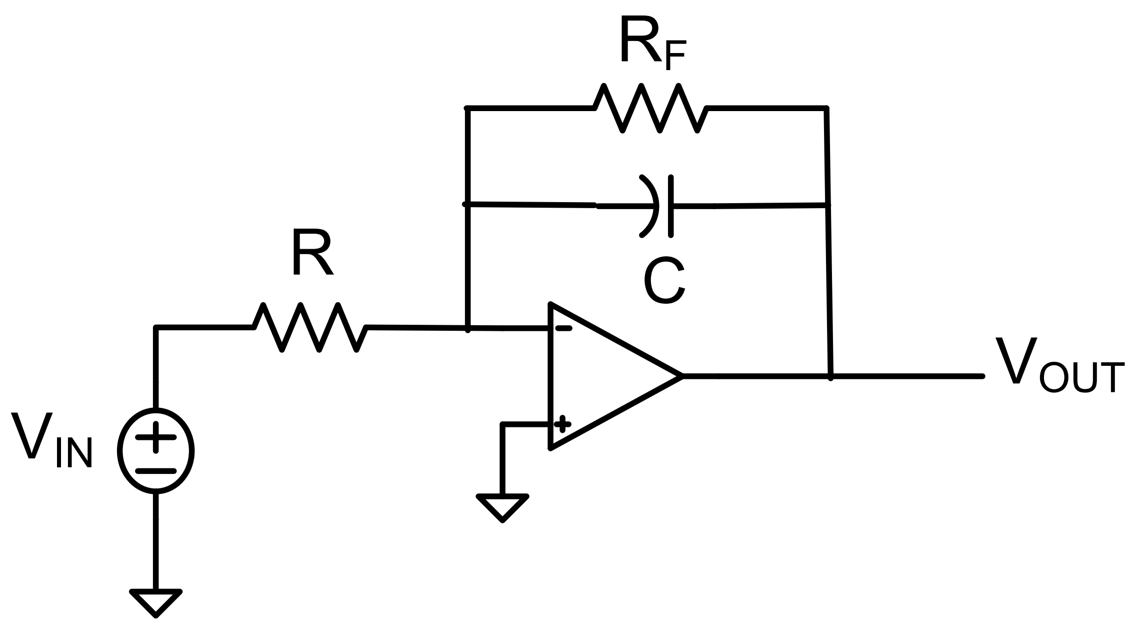
Obtained from inverting integrator by preceding or following with inverter

Requires more components

Also widely used

Same issues affect noninverting integrator

Lossy Integrator



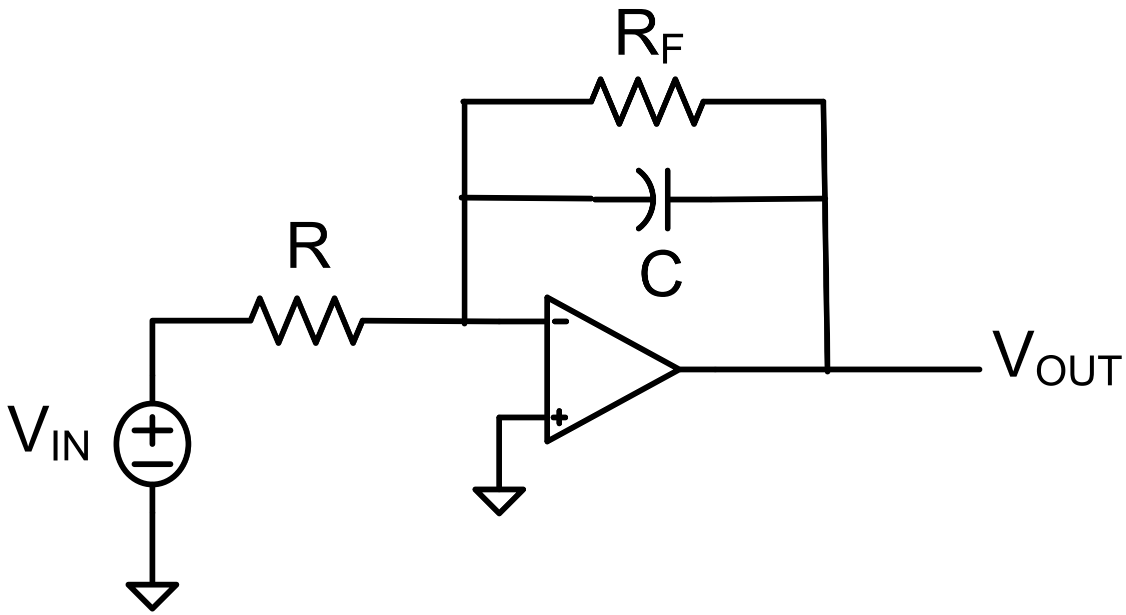
Add a large resistor to slowly drain charge off of C and prevent divergence

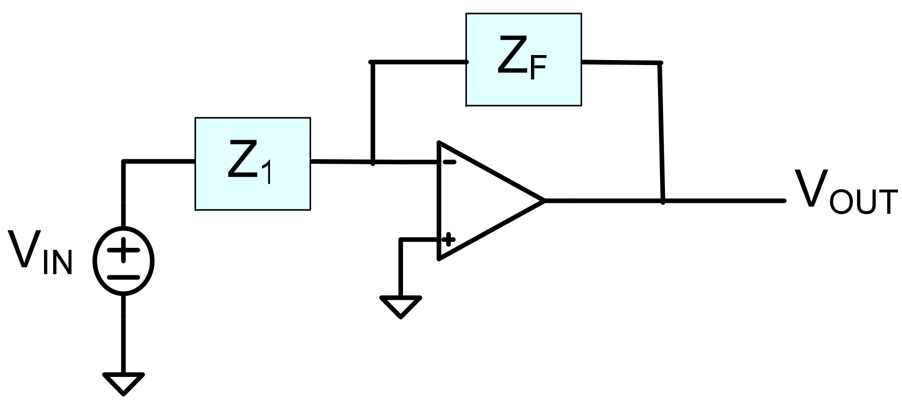
Allows integrator to be used "open-loop"

Changes the dc gain from -∞ to $-R_F/R$

But the lossy integrator is no longer a perfect integrator

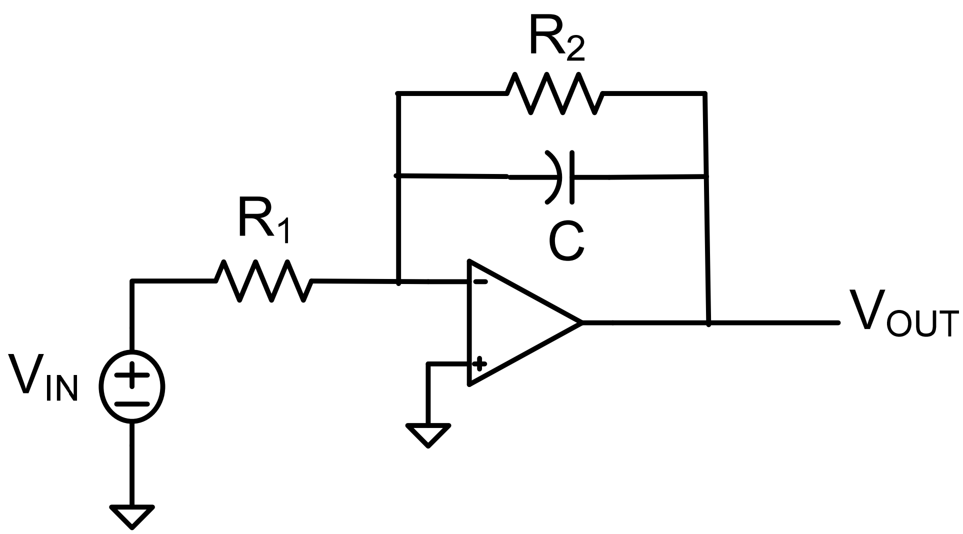
What if R_F is not so large?





$$T(s) = -\frac{Z_F(s)}{Z_1(s)}$$

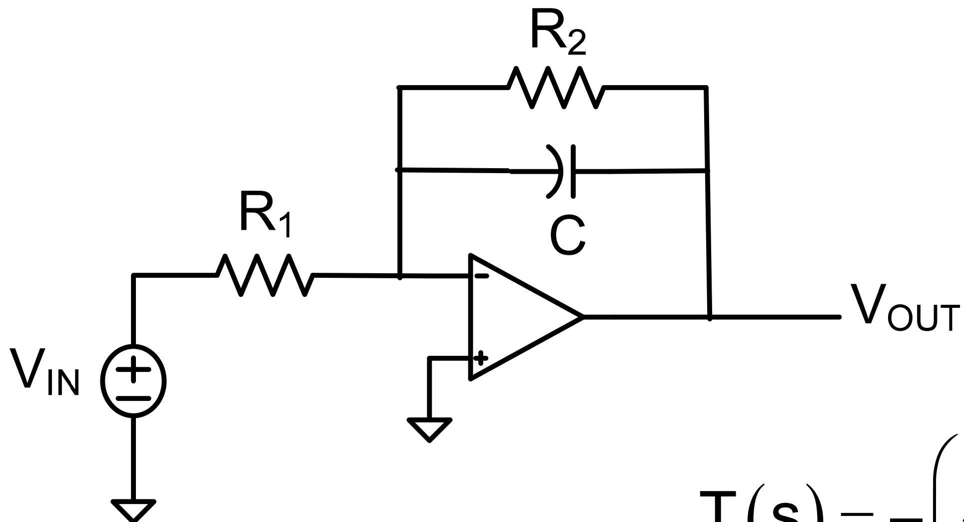
What if R_F is not so large?



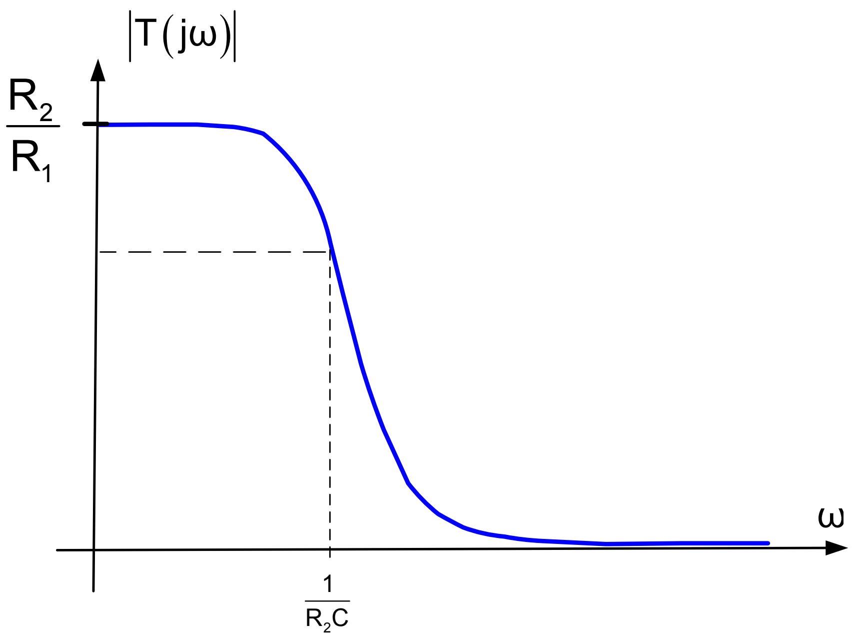
$$T(s) = -\frac{Z_F(s)}{Z_1(s)}$$

$$T(s) = -\frac{\left(\dfrac{R_2 \dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}}\right)}{R_1} = -\left(\frac{R_2}{R_1}\right)\frac{1}{1+sCR_2}$$

First-order lowpass filter with a dc gain of R_2/R_1



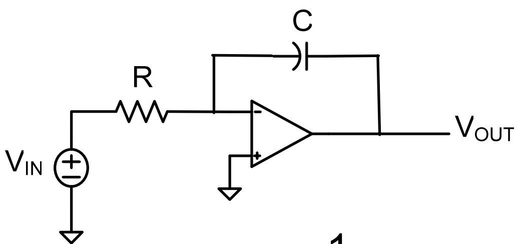
$$T(s) = -\left(\frac{R_2}{R_1}\right)\frac{1}{1+sCR_2}$$



R_2 controls the pole
(and also the dc gain)

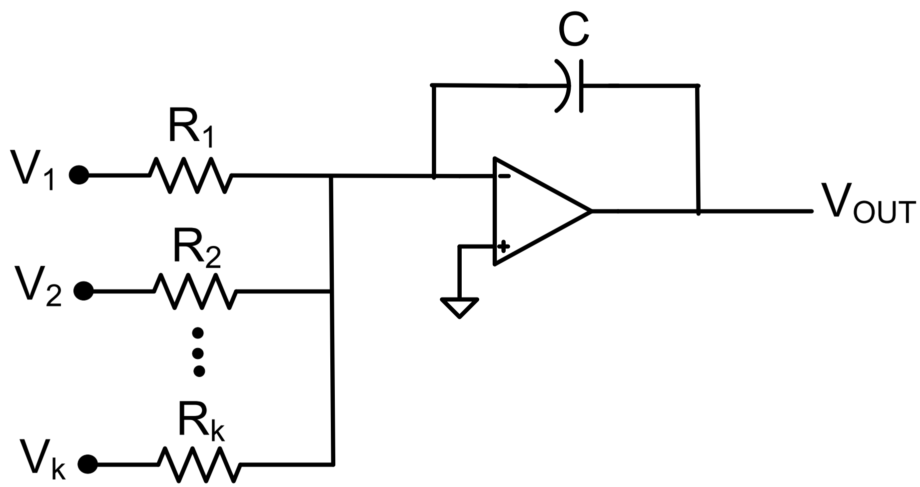
R_1 controls the dc gain
(and not the pole)

Summing Integrator



$$T(s) = -\frac{1}{sRC}$$

$$V_{OUT} = -\frac{1}{sRC} V_{IN}$$

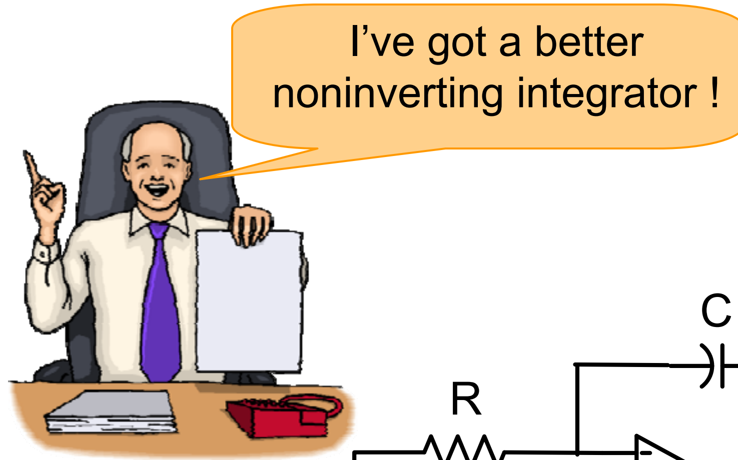


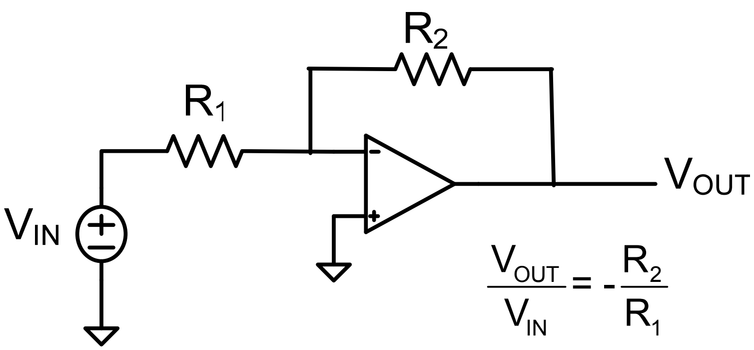
By superposition

$$V_{OUT} = -\frac{1}{sR_1C}V_1 - \frac{1}{sR_2C}V_2 - ..... - \frac{1}{sR_kC}V_k$$

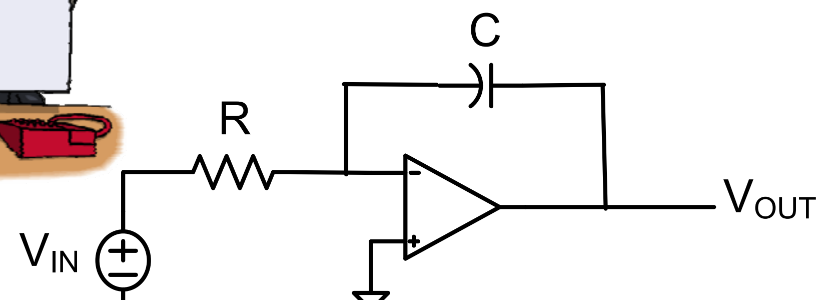
- All inverting functions
- Can have any number of inputs
- Weights independently controlled by resistor values
- Weights all changed by C

$$V_{OUT} = -\sum_{i=1}^{k} \frac{1}{sR_iC} V_i$$

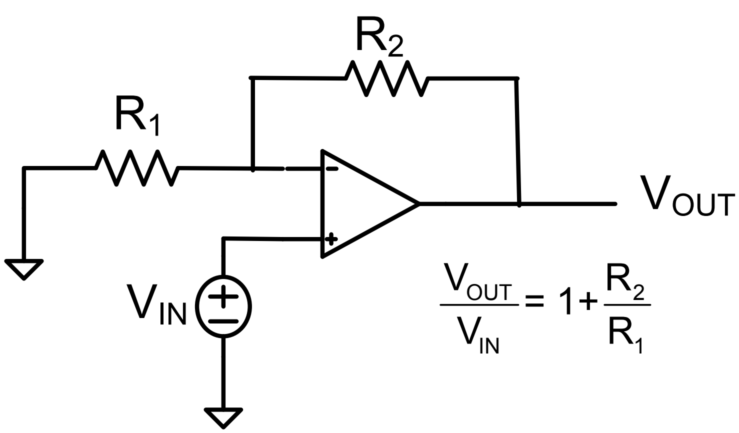




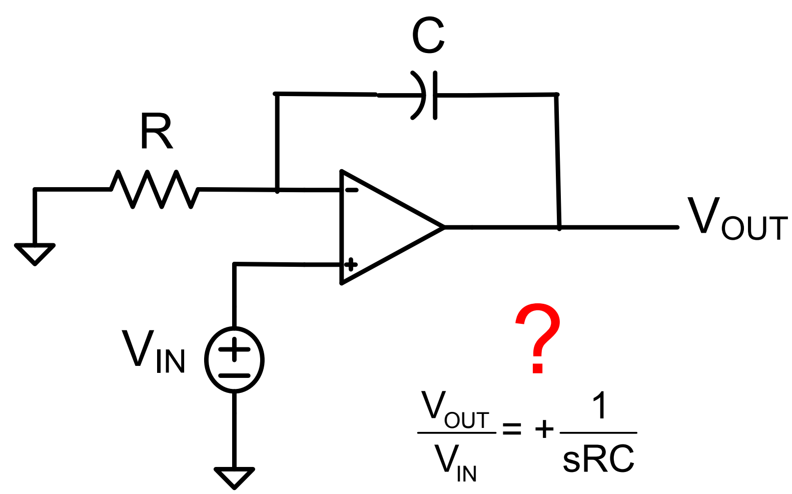
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}$$



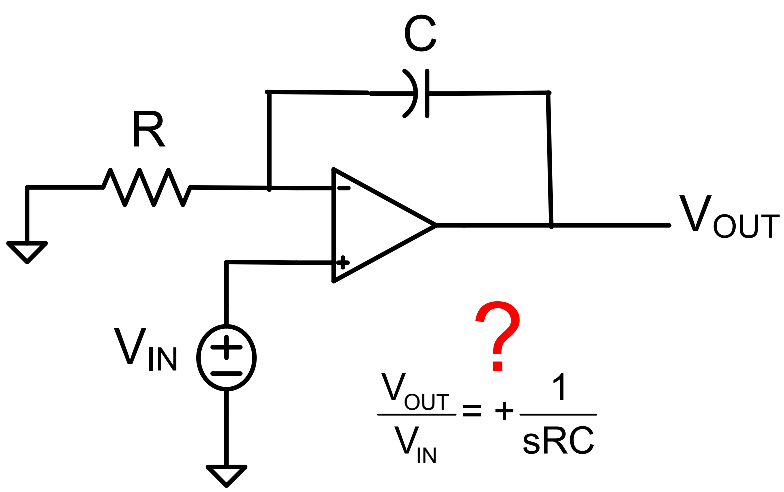
$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{sRC}$$



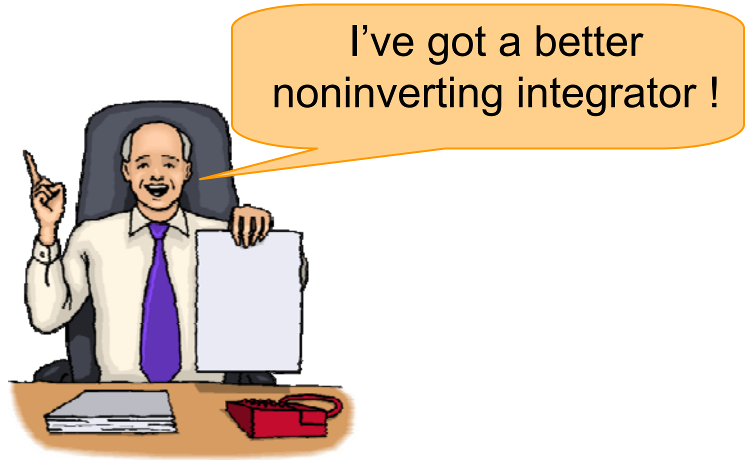
$$\frac{V_{OUT}}{V_{IN}} = 1+\frac{R_2}{R_1}$$

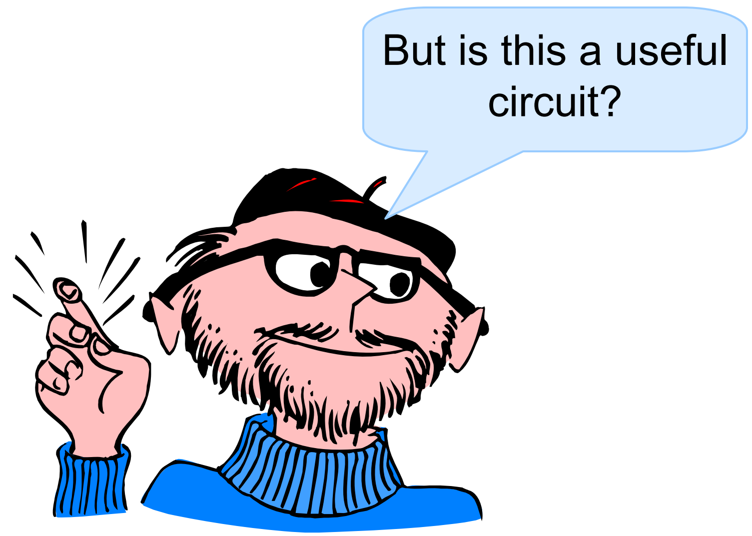


$$\frac{V_{OUT}}{V_{IN}} = +\frac{1}{sRC}$$



$$\frac{V_{OUT}}{V_{IN}} \overset{?}{=} +\frac{1}{sRC}$$





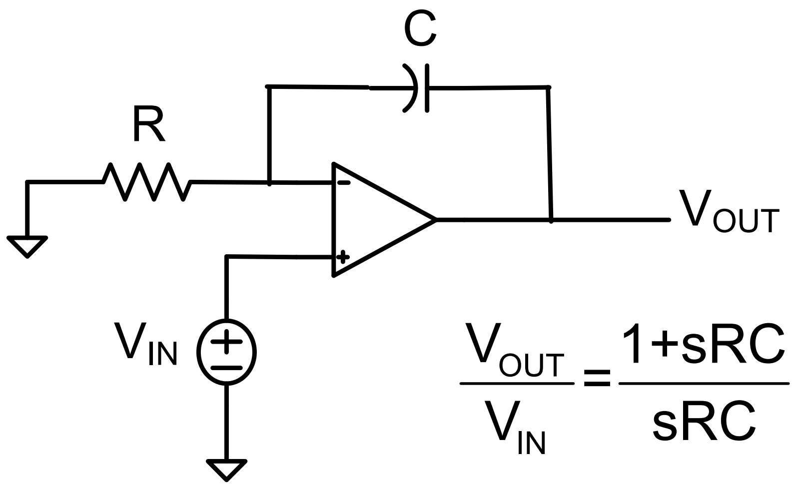
$$V_{IN}\left(sC+G\right)=V_{OUT}sC$$

$$\frac{V_{OUT}}{V_{IN}}=\frac{1+sRC}{sRC}$$

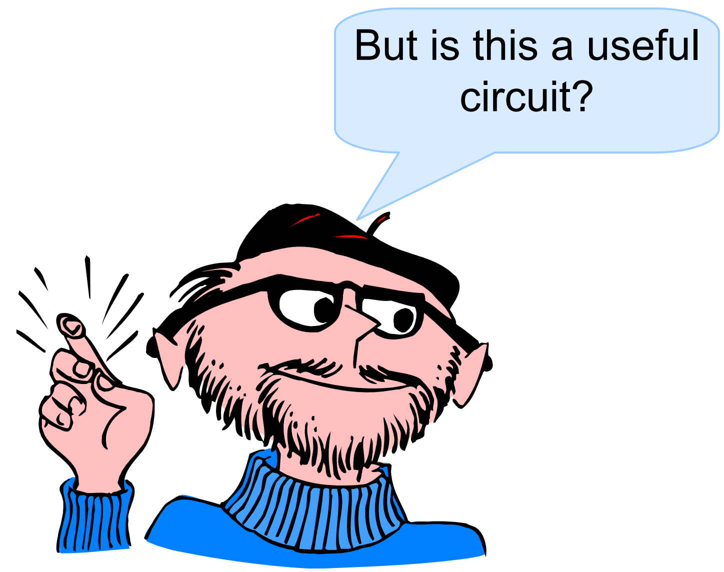
It has a noninverting transfer function

But it is not an noninverting integrator !

First-Order Highpass Filter

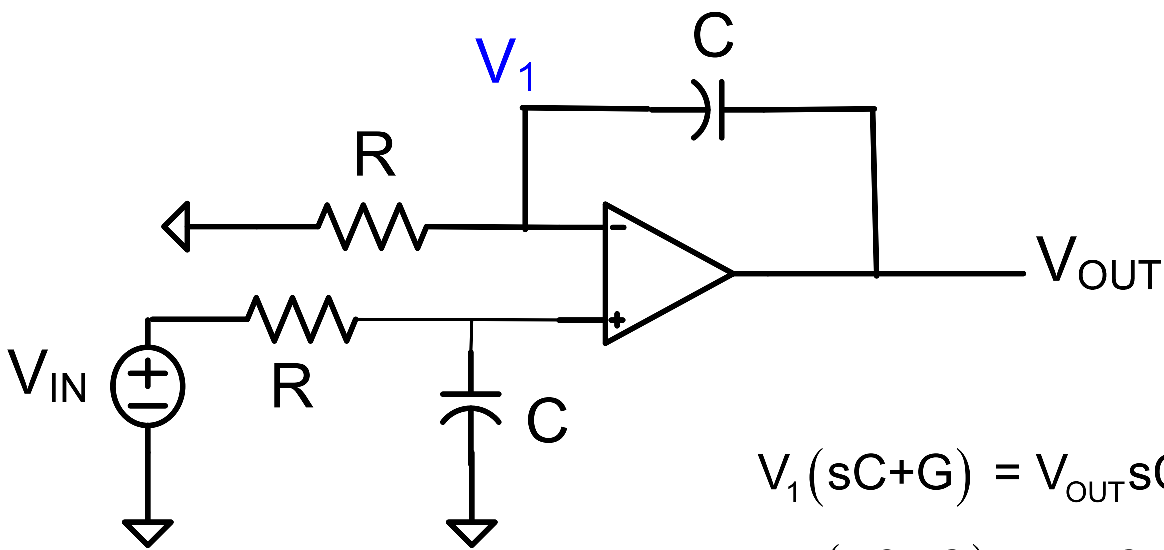


$$\frac{V_{OUT}}{V_{IN}} = \frac{1+sRC}{sRC}$$



This is a first-order high-pass amplifier (or filter) but the gain at dc goes to ∞ so applications probably limited.

Two-capacitor noninverting integrator



$$V_1(sC+G) = V_{OUT}sC$$

$$V_1(sC+G) = V_{IN}G$$

$$\frac{V_{OUT}}{V_{IN}} = +\frac{1}{sRC}$$

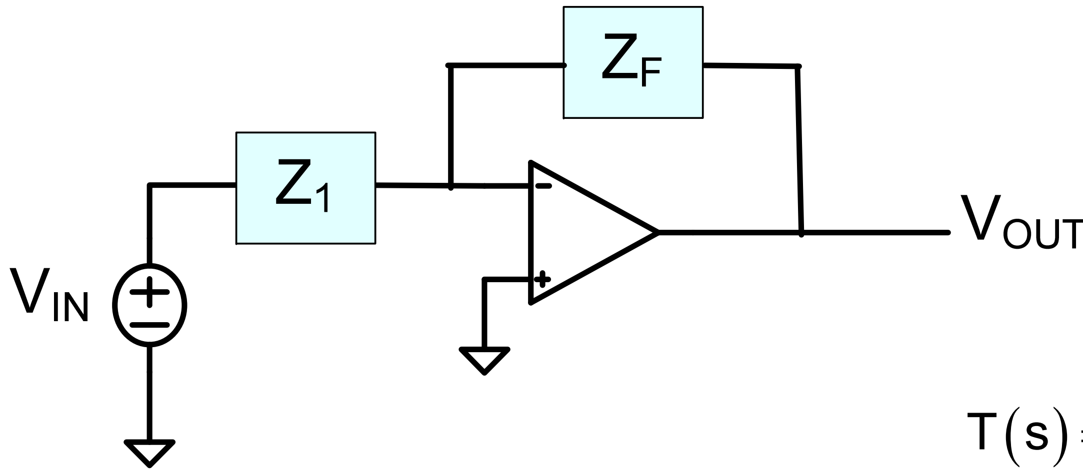
Requires matched resistors and matched capacitors

Actually uses a concept called “pole-zero cancellation”

Generally less practical than the cascade with an inverter

Generalized Inverting Amplifier

s-domain representation

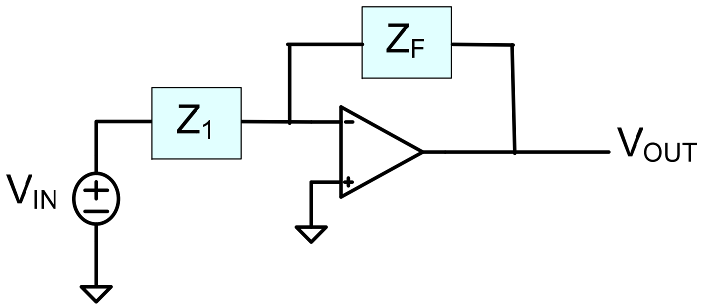


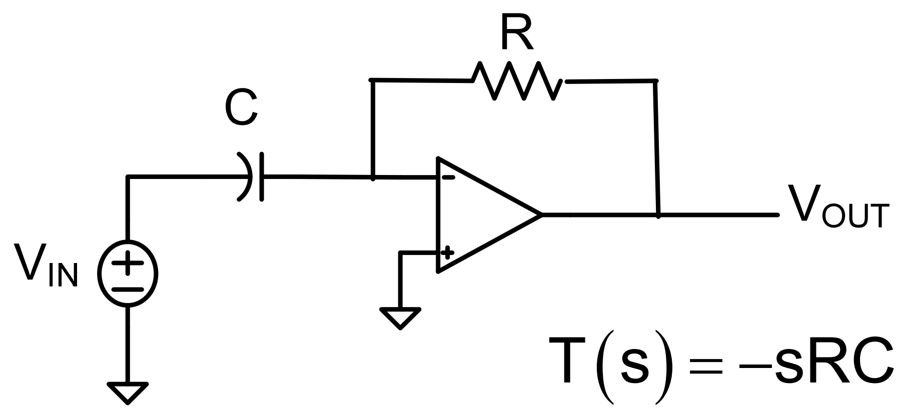
$$T(s) = -\frac{Z_F(s)}{Z_1(s)}$$

If Z_1=1/sC, Z_F=R, obtain $$T(s) = -sRC$$

What is this circuit?

Generalized Inverting Amplifier





$$T(s) = -sRC$$

What is this circuit?

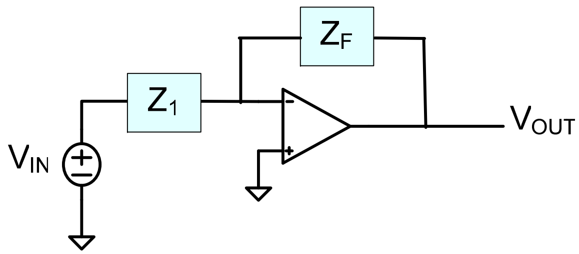
Consider the differential equation $y = K\dfrac{dx(t)}{dt}$

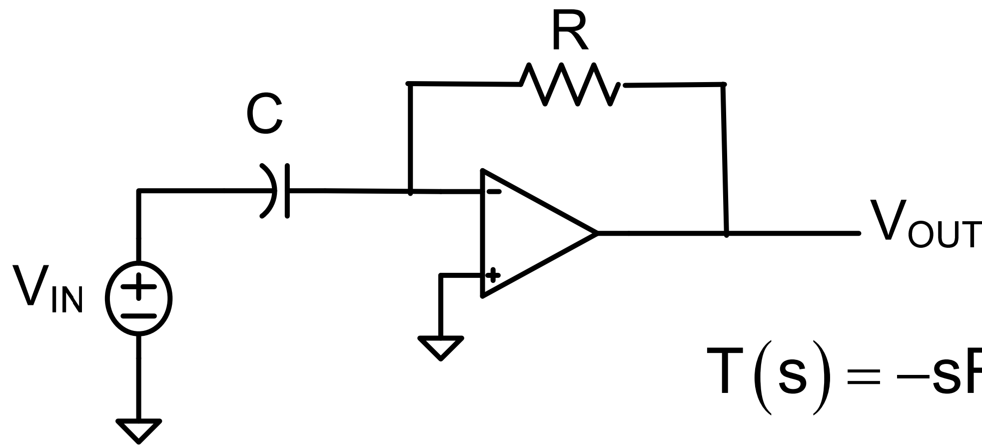
Taking the Laplace Transform, obtain $Y(s) = Ks\,X(s)$ $\qquad T(s) = \dfrac{Y(s)}{X(s)} = Ks$

K^{-1} is the frequency where $|T(j\omega)|=1$

Thus, this circuit is an <u>inverting</u> differentiator with a unity gain frequency of $K^{-1} = (RC)^{-1}$

Inverting Differentiator





$$T(s) = -sRC$$

Differentiator gain ideally goes to ∞ at high frequencies

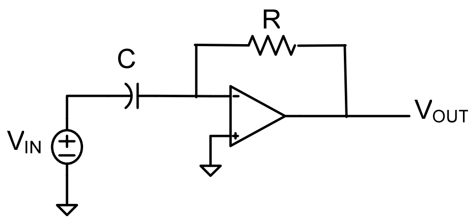
Differentiator not widely used

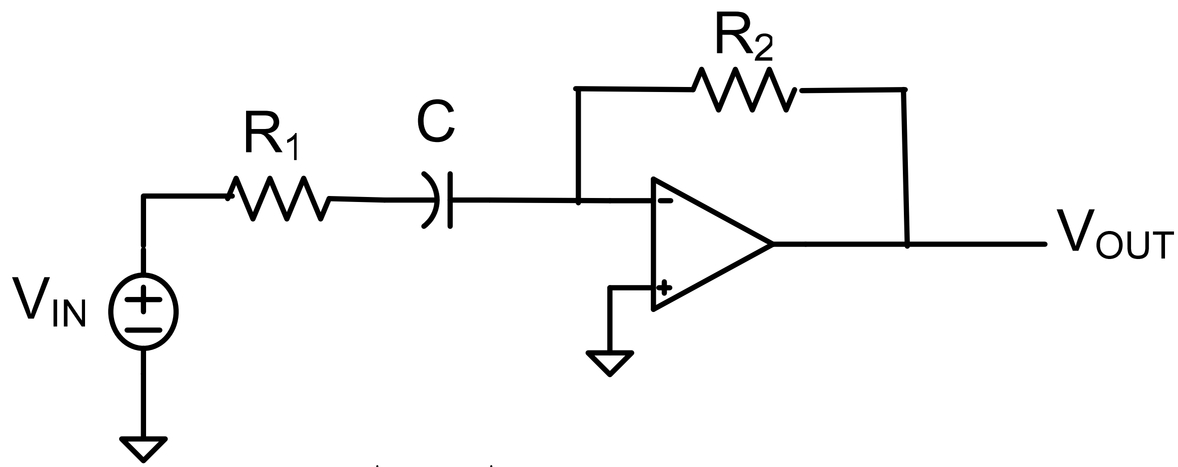
Differentiator relentlessly amplifies noise

Stability problems with implementation (not discussed here)

Placing a resistor in series with C will result in a lossy differentiator that has some applications

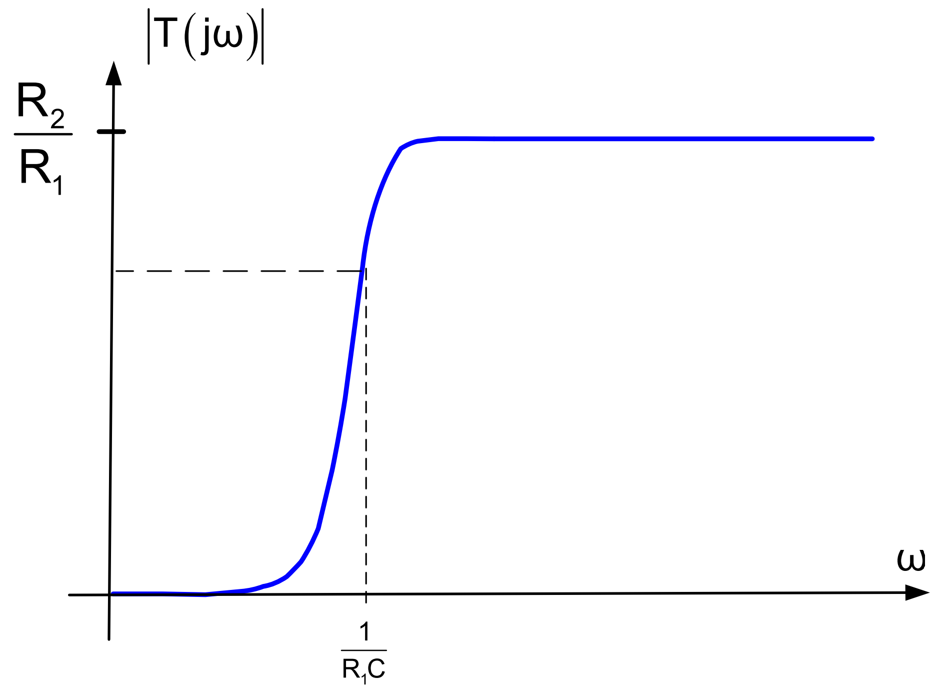
First-order High-pass Filter



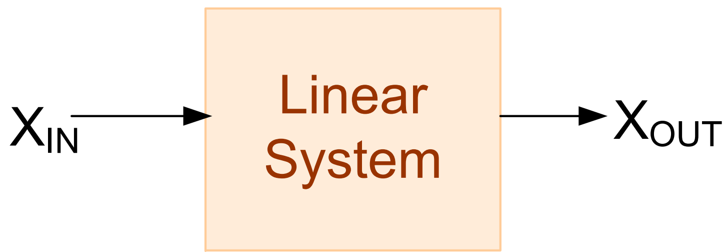


$$T(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{sR_2C}{1+R_1Cs}$$

$$\left|T(j\omega)\right| = \frac{\omega R_2 C}{\sqrt{1+\left(\omega R_1 C\right)^2}}$$



Applications of integrators to solving differential equations



Standard Integral form of a differential equation

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

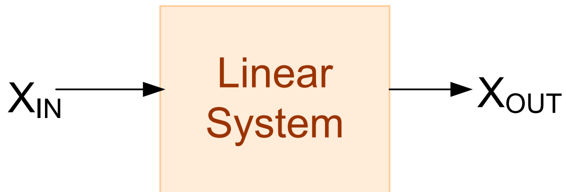
Standard differential form of a differential equation

$$X_{OUT} = \alpha_1 X'_{OUT} + \alpha_2 X''_{OUT} + \alpha_3 X'''_{OUT} + \ldots + \beta_1 X_{IN} + \beta_2 X'_{IN} + \beta_3 X''_{IN} + \ldots$$

Initial conditions not shown

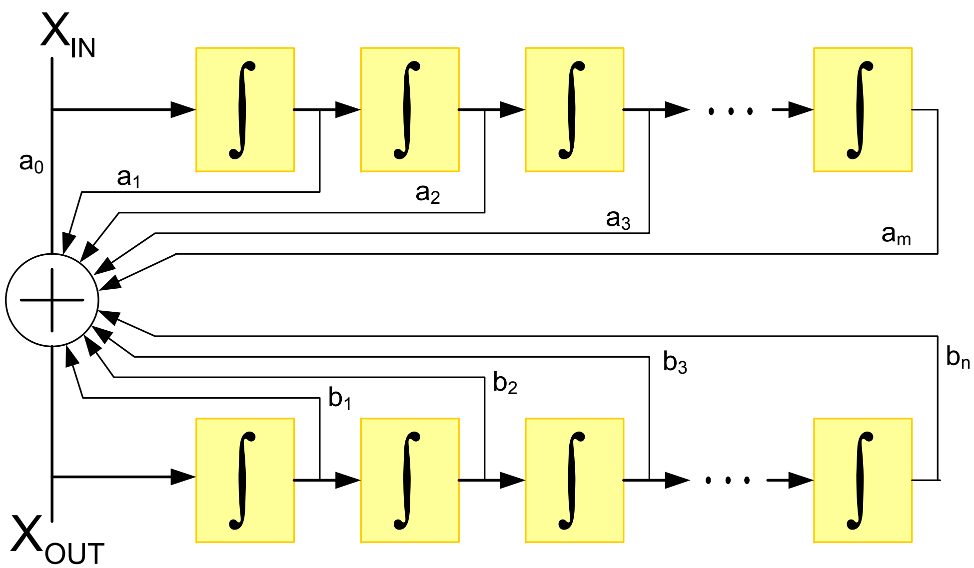
Can express any system in either differential or integral form

Applications of integrators to solving differential equations



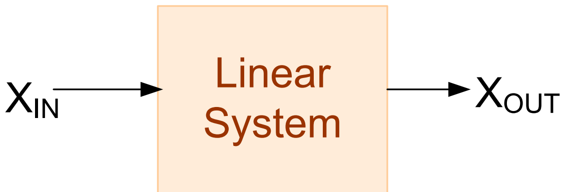
Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$



This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past

Applications of integrators to solving differential equations



Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

Take the Laplace transform of this equation

$$\mathcal{X}_{OUT} = b_1 \frac{1}{s}\mathcal{X}_{OUT} + b_2 \frac{1}{s^2}\mathcal{X}_{OUT} + b_3 \frac{1}{s^3}\mathcal{X}_{OUT} + \ldots + b_n \frac{1}{s^n} + a_0 \mathcal{X}_{IN} + a_1 \frac{1}{s}\mathcal{X}_{IN} + a_2 \frac{1}{s^2}\mathcal{X}_{IN} + a_3 \frac{1}{s^3}\mathcal{X}_{IN} + \ldots + a_m \frac{1}{s^m}$$

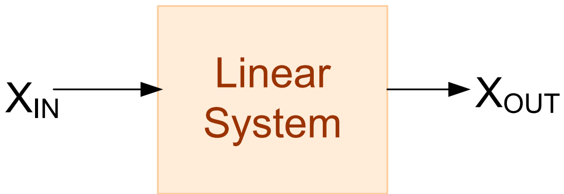
Multiply by s^n and assume m=n   (some of the coefficients can be 0)

$$s^n \mathcal{X}_{OUT} = b_1 s^{n-1} \mathcal{X}_{OUT} + b_2 s^{n-2} \mathcal{X}_{OUT} + b_3 s^{n-3} \mathcal{X}_{OUT} + \ldots + b_n + a_0 s^n \mathcal{X}_{IN} + a_1 s^{n-1} \mathcal{X}_{IN} + a_2 s^{n-2} \mathcal{X}_{IN} + a_3 s^{n-3} \mathcal{X}_{IN} + \ldots + a_n$$

$$\mathcal{X}_{OUT}\left(s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n\right) = \mathcal{X}_{IN}\left(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n\right)$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n}$$

Applications of integrators to solving differential equations



Consider the standard integral form

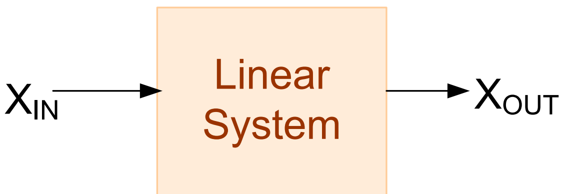
$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 \mathrm{s}^{n} + a_1 \mathrm{s}^{n-1} + a_2 \mathrm{s}^{n-2} + a_3 \mathrm{s}^{n-3} + \ldots + a_n}{\mathrm{s}^{n} - b_1 \mathrm{s}^{n-1} - b_2 \mathrm{s}^{n-2} - b_3 \mathrm{s}^{n-3} - \ldots - b_n}$$

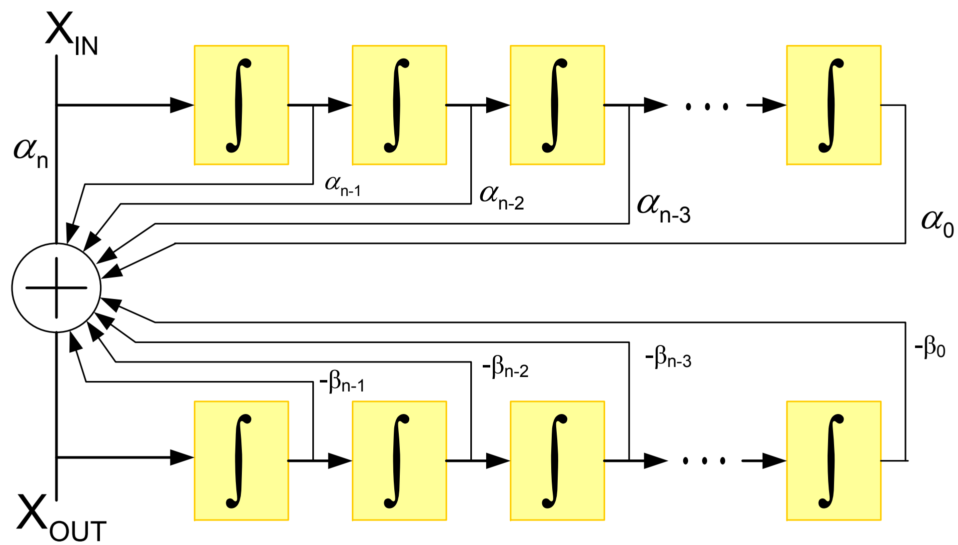
This can be written in more standard form

$$T(s) = \frac{\alpha_n \mathrm{s}^{n} + \alpha_{n-1} \mathrm{s}^{n-1} + \ldots \alpha_1 \mathrm{s} + \alpha_0}{\mathrm{s}^{n} + \beta_{n-1} \mathrm{s}^{n-1} + \ldots + \beta_1 \mathrm{s} + \beta_0}$$

Applications of integrators to filter design



$$T(s)=\frac{\alpha_n s^n+\alpha_{m-1}s^{n-1}+\ldots\alpha_1 s+\alpha_0}{s^n+\beta_{n-1}s^{n-1}+\ldots+\beta_1 s+\beta_0}$$



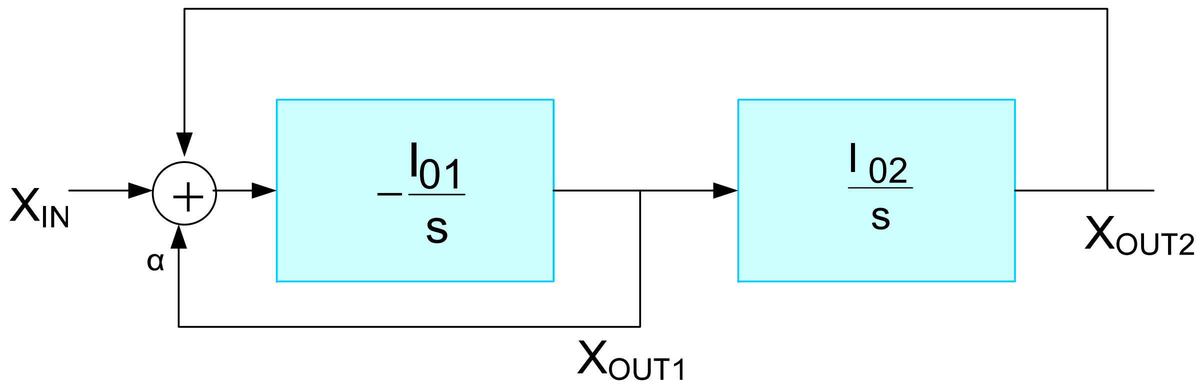
Can design (synthesize) any T(s) with just integrators and summers !

Integrators are not used “open loop” so loss is not added

Although this approach to filter design works, often more practical methods are used

End of Lecture 12

Applications of integrators to filter design



This is a two-integrator-loop filter

$$X_{OUT1}=\left(-\frac{I_{01}}{s}\right)\left(X_{IN}+X_{OUT2}+\alpha X_{OUT1}\right)$$

$$X_{OUT2}=\left(\frac{I_{02}}{s}\right)X_{OUT1}$$

$$\frac{X_{OUT1}}{X_{IN}}=T_1(s)=\frac{-I_{01}s}{s^2+\alpha I_{01}s+I_{01}I_{02}}$$

$$\frac{X_{OUT2}}{X_{IN}}=T_2(s)=\frac{-I_{01}I_{02}}{s^2+\alpha I_{01}s+I_{01}I_{02}}$$

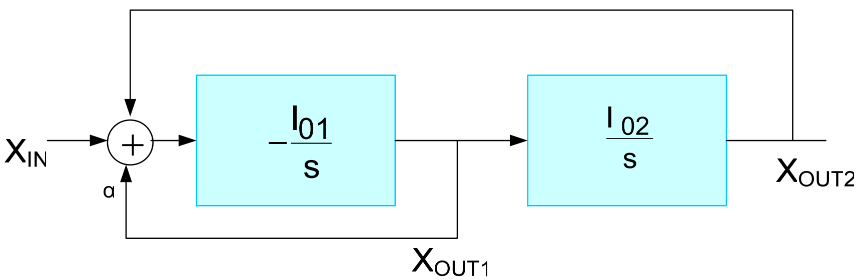
These are 2-nd order filters

If $I_{01}=I_{02}=I_0$, these transfer functions reduce to

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$

$$T_2(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Applications of integrators to filter design



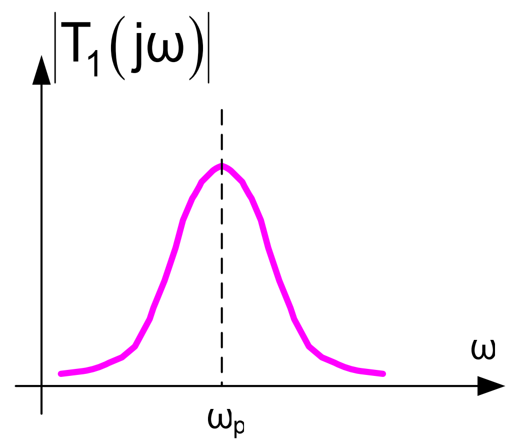
$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$

$$T_2(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Consider $T_1(j\omega)$

$$T_1(j\omega)=\frac{-j\omega I_0}{\left(I_0^2-\omega^2\right)+j\omega\alpha I_0}$$

$$\left|T_1(j\omega)\right|=\frac{\omega I_0}{\sqrt{\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2}}$$



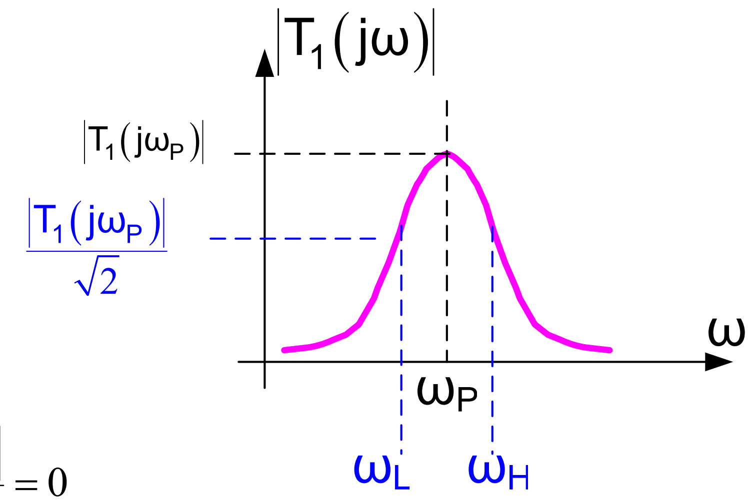
This is the standard 2[nd] order bandpass transfer function

Now lets determine the BW and ω_P

Applications of integrators to filter design

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$



$$|T_1(j\omega)| = \frac{\omega I_0}{\sqrt{\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}}$$

To determine ω_P, must set $\frac{d|T_1(j\omega)|}{d\omega} = 0$

This will occur also when $\frac{d|T_1(j\omega)|^2}{d\omega^2} = 0$ and the latter is easier to work with

$$|T_1(j\omega)|^2 = \frac{\omega^2 I_0^2}{\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}$$

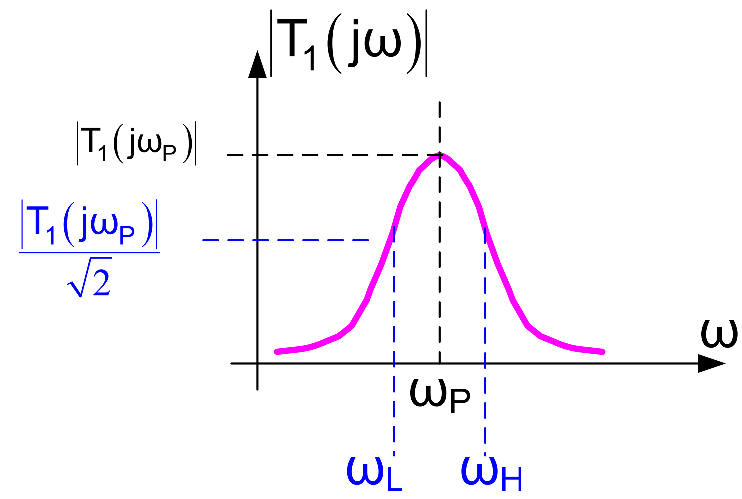
$$\frac{d|T_1(j\omega)|^2}{d\omega^2} = \frac{\left(\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right) I_0^2 - \omega^2 I_0^2 \left(-2\left(I_0^2 - \omega^2\right) + \left(\alpha I_0\right)^2\right)}{\left[\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right]^2} = 0$$

Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2} \qquad \left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2-\omega^2\right)^2 + (\omega \alpha I_0)^2}}$$



$$\frac{d\left|T_1(j\omega)\right|^2}{d\omega^2} = \frac{\left(\left(I_0^2-\omega^2\right)^2 + (\omega\alpha I_0)^2\right) I_0^2 - \omega^2 I_0^2\left(-2\left(I_0^2-\omega^2\right) + (\alpha I_0)^2\right)}{\left[\left(I_0^2-\omega^2\right)^2 + (\omega\alpha I_0)^2\right]^2} = 0$$

It suffices to set the numerator to 0

$$\left(\left(I_0^2-\omega^2\right)^2 + (\omega\alpha I_0)^2\right) I_0^2 = \omega^2 I_0^2\left(-2\left(I_0^2-\omega^2\right) + (\alpha I_0)^2\right)$$

Solving, we obtain

$$\omega_P = I_0$$

Substituting back into the magnitude expression, we obtain

$$\left|T_1(j\omega_P)\right| = \frac{I_0 I_0}{\sqrt{\left(I_0^2 - I_0^2\right) + (I_0\alpha)^2}} = \frac{1}{\alpha}$$

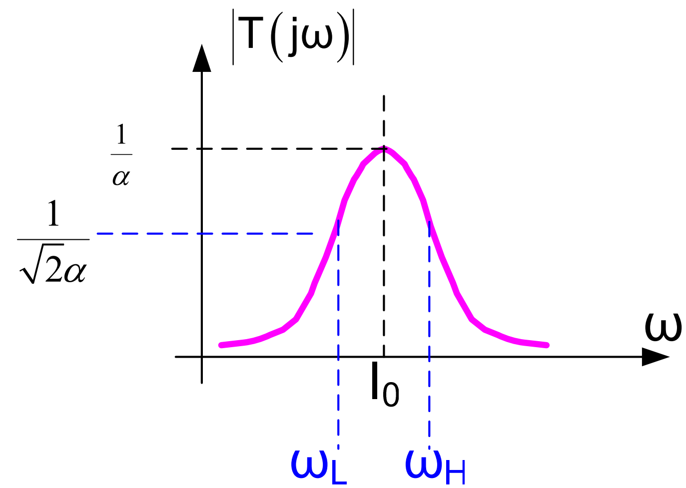
Although the analysis is somewhat tedious, the results are clean

Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2} \qquad \left|T_1(j\omega)\right|=\frac{\omega I_0}{\sqrt{\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2}}$$



To obtain ω_L and ω_H, must solve $\left|T_1(j\omega)\right|=\frac{1}{\sqrt{2}\alpha}$

This becomes

$$\frac{1}{2\alpha^2}=\frac{\left(\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2\right)I_0^2-\omega^2 I_0^2\left(-2\left(I_0^2-\omega^2\right)+\left(\alpha I_0\right)^2\right)}{\left[\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2\right]^2}$$

The expressions for ω_L and ω_H can be easily obtained but are somewhat messy, but from these expressions, we obtain the simple expressions

$$BW = \omega_H - \omega_L = \alpha I_0$$

$$\sqrt{\omega_H\omega_L} = I_0$$

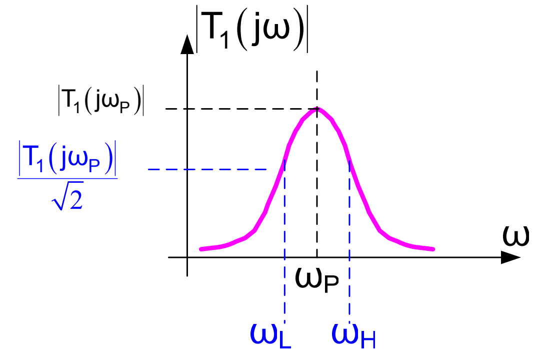
Applications of integrators to filter design

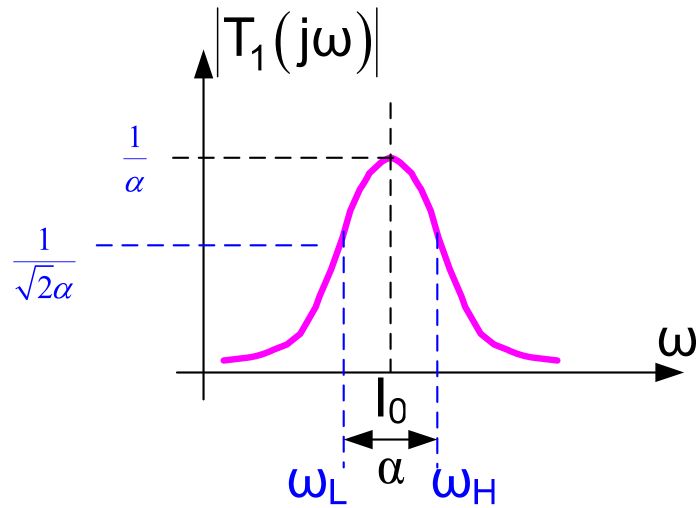
The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2} \qquad \left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}}$$

$$\omega_P = I_0 \qquad \left|T_1(j\omega_P)\right| = \frac{1}{\alpha}$$





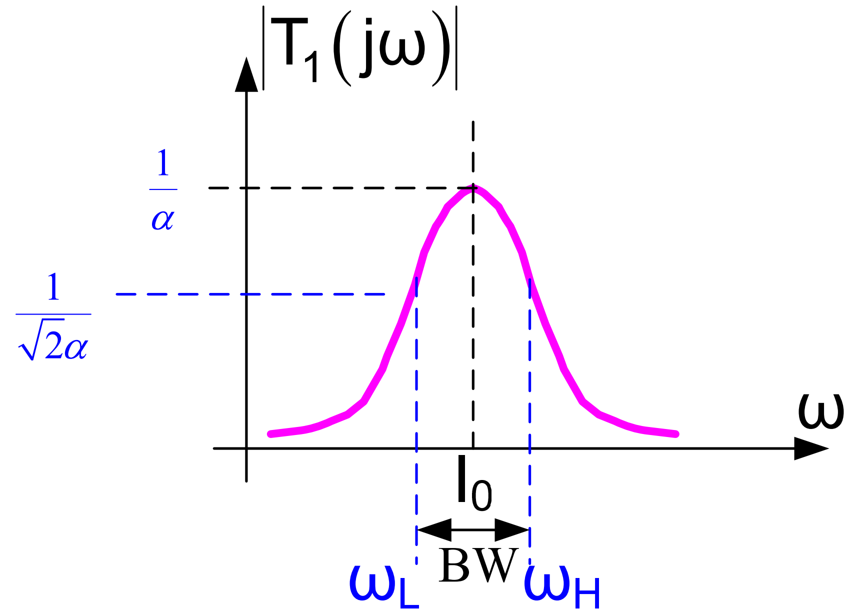
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$

$$BW = \alpha I_0 \qquad \sqrt{\omega_H \omega_L} = I_0$$



Often express the standard 2nd order bandpass transfer function as

$$T_1(s)=\frac{-I_0 s}{s^2+BWs+I_0^2}$$

Applications of integrators to filter design

The 2nd order Bandpass Filter

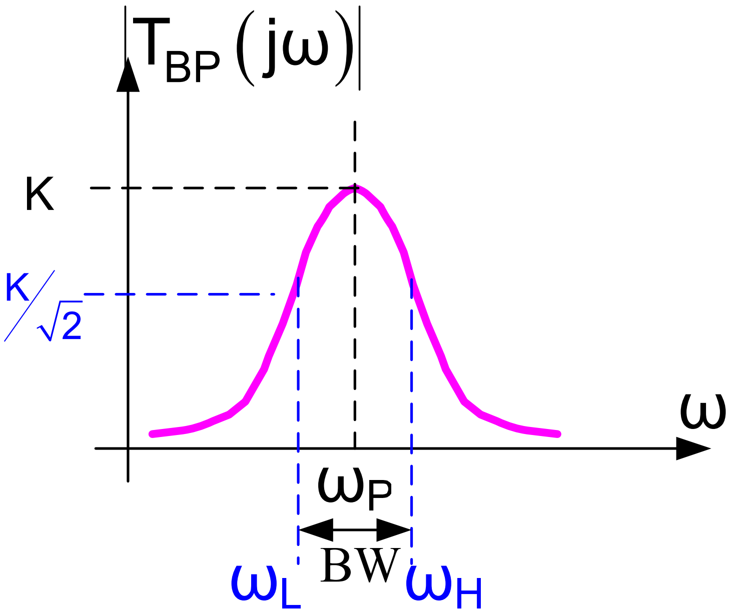
These results can be generalized

$$T_{BP}(s) = \frac{Hs}{s^2+as+b}$$

$$BW = a$$

$$\omega_P = \sqrt{b}$$

$$K = \frac{|H|}{a}$$



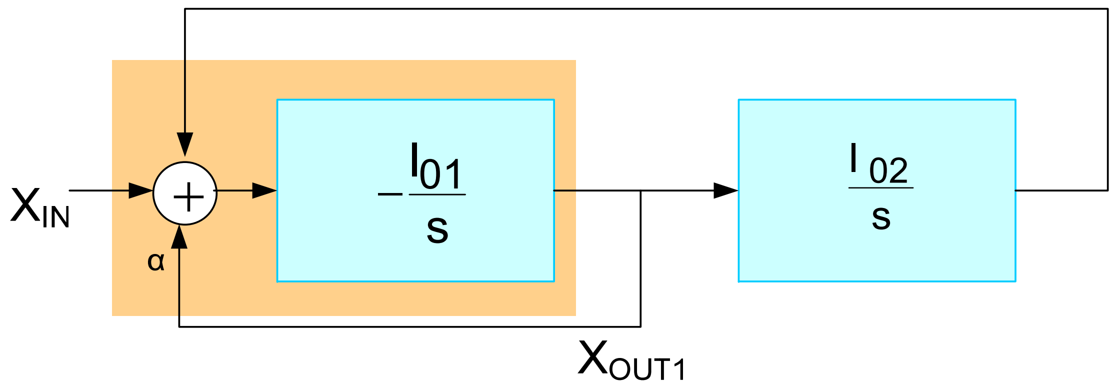
Applications of integrators to filter design

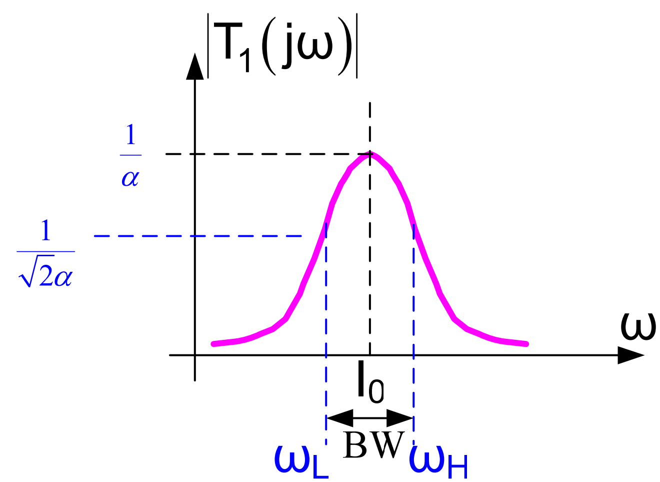
The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$T_1(s) = \frac{-I_0 s}{s^2 + BWs + I_0^2}$$





Can readily be implemented with a summing inverting integrator and a noninverting integrator

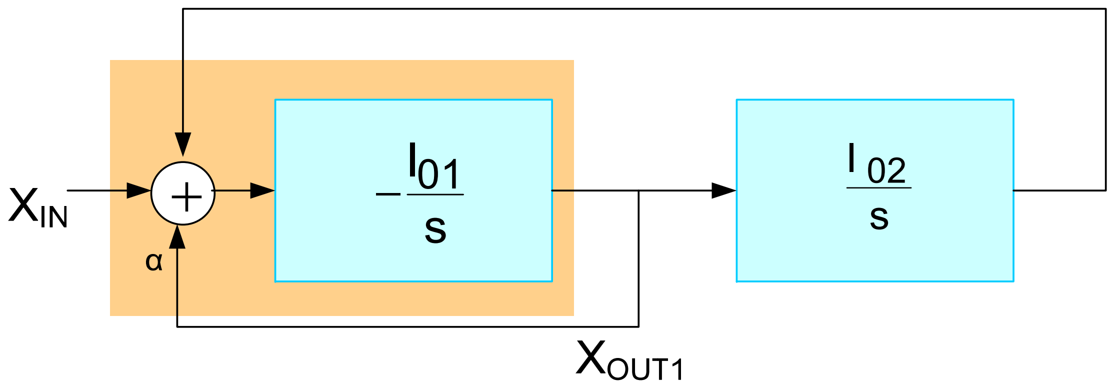
Applications of integrators to filter design

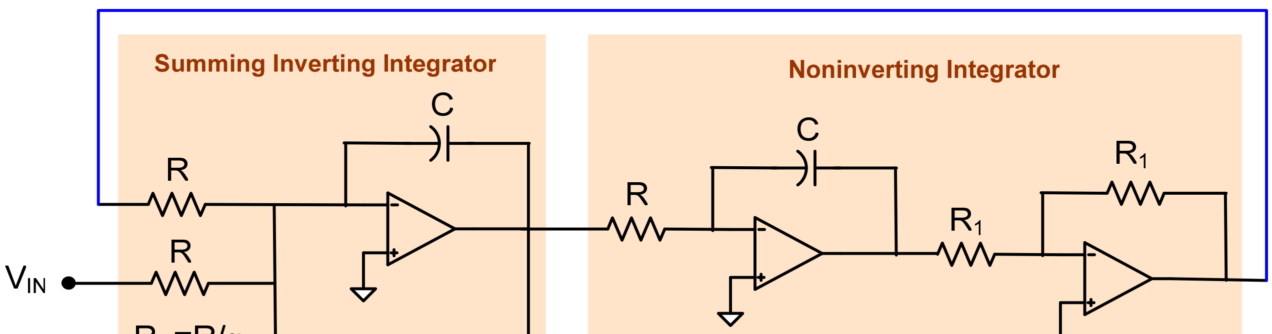
The 2nd order Bandpass Filter

Determine the BW and ω_P

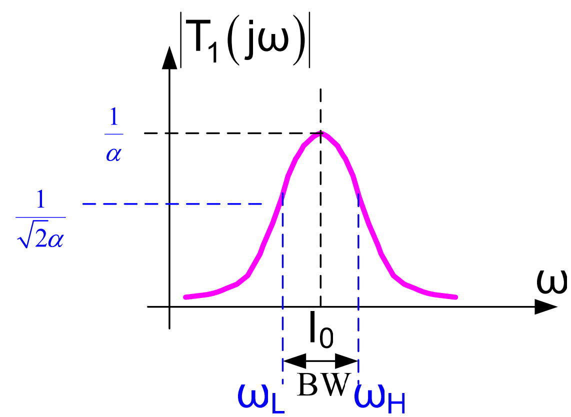
$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$T_1(s) = \frac{-I_0 s}{s^2 + BWs + I_0^2}$$





$$I_0 = \frac{1}{RC} \qquad \therefore BW = \frac{\alpha}{RC}$$



$$\omega_P = I_0$$

$$BW = \alpha I_0$$

- Widely used 2nd order Bandpass Filter
- BW can be adjusted with R_Q
- Peak gain changes with R_Q
- Note no loss is added to the integrators

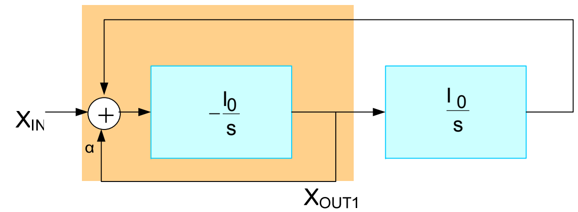
Applications of integrators to filter design

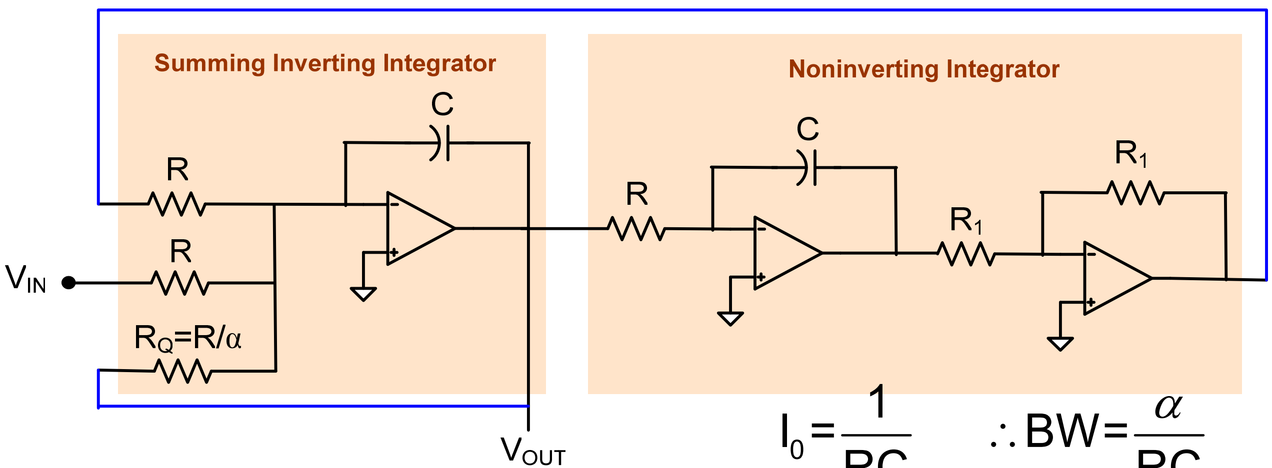
The 2nd order Bandpass Filter

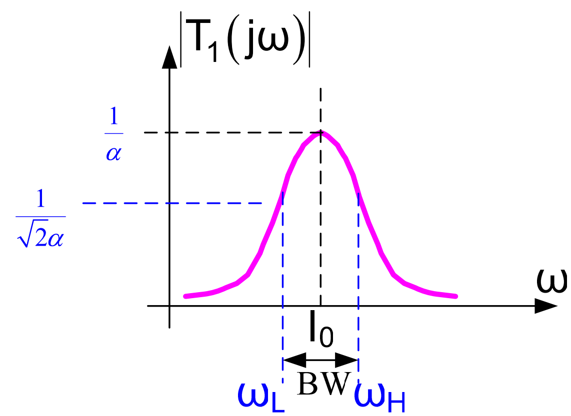
Design Strategy

Assume BW and ω_P are specified

$$T_{BP}(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$







$$I_0=\frac{1}{RC} \qquad \therefore BW=\frac{\alpha}{RC}$$

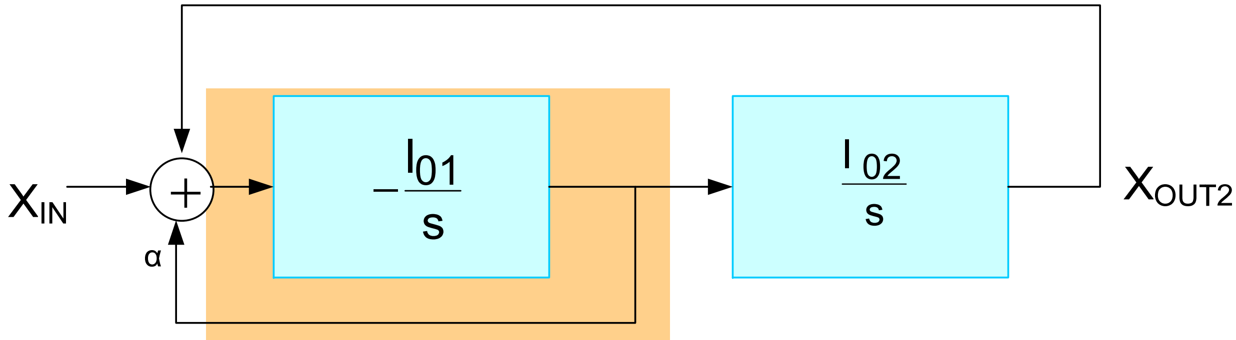
$$\omega_P = I_0$$

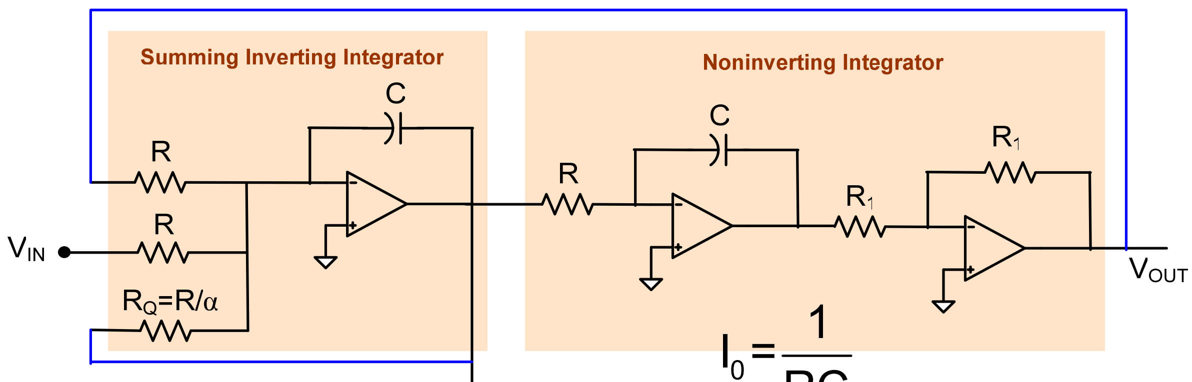
1. Pick C (use some practical or convenient value)
2. Solve expression $\omega_P=\dfrac{1}{RC}$ to obtain R
3. Solve expression $BW=\dfrac{\alpha}{RC}$ to obtain α and thus R_Q

Applications of integrators to filter design

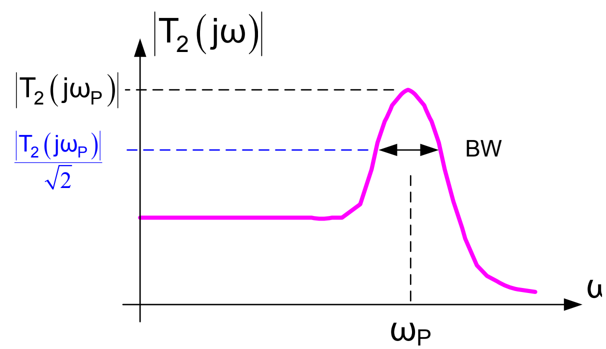
The 2nd order Lowpass Filter

$$T_2(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$





$$I_0 = \frac{1}{RC}$$

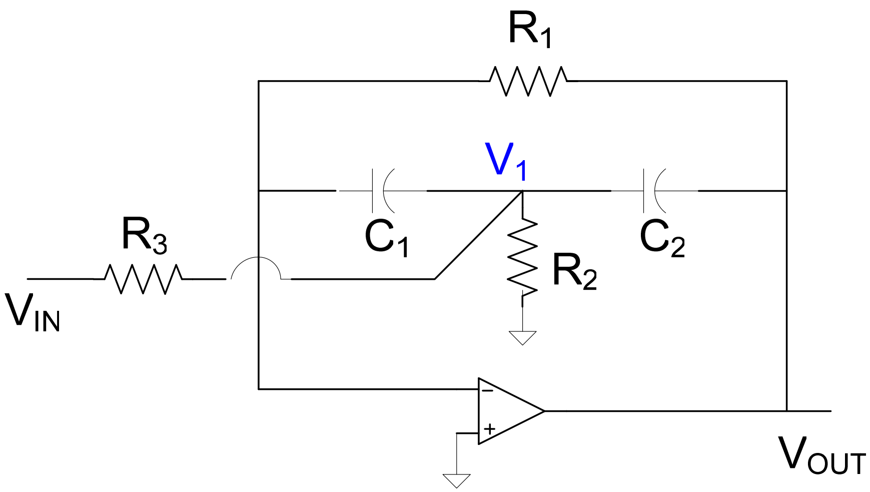


Exact expressions for BW and ω_P are very complicated but $\omega_P \approx I_0$

- Widely used 2nd order Lowpass Filter
- BW can be adjusted with R_Q but expression not so simple
- Peak gain changes with R_Q
- Note no loss is added to the integrators

Design procedure to realize a given 2nd order lowpass function is straightforward

Another 2nd-order Bandpass Filter



$$V_1\left(sC_1+sC_2+G_2+G_3\right) = V_{OUT}sC_2+V_{IN}G_3$$

$$V_1sC_1+V_{OUT}G_1 = 0$$

$$T(s) = -\frac{\frac{s}{R_3C_2}}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_1C_2}\right)+\frac{1}{(R_2//R_3)R_1C_1C_2}}$$

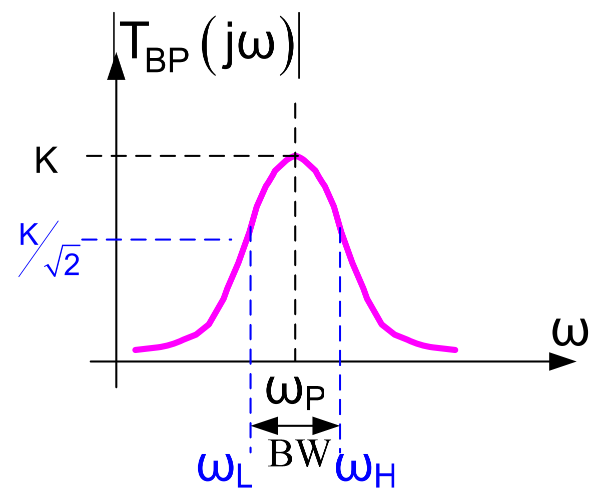
If the capacitors are matched and equal to C

$$T(s) = -\frac{\frac{s}{R_3C}}{s^2+s\left(\frac{2}{R_1C}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

Since this is of the general form of a 2nd order BP transfer function, obtain

$$\omega_P=\frac{1}{\sqrt{R_1\left(R_2//R_3\right)}C}$$

$$BW = \frac{2}{R_1C} \qquad K=\frac{R_1}{2R_3}$$

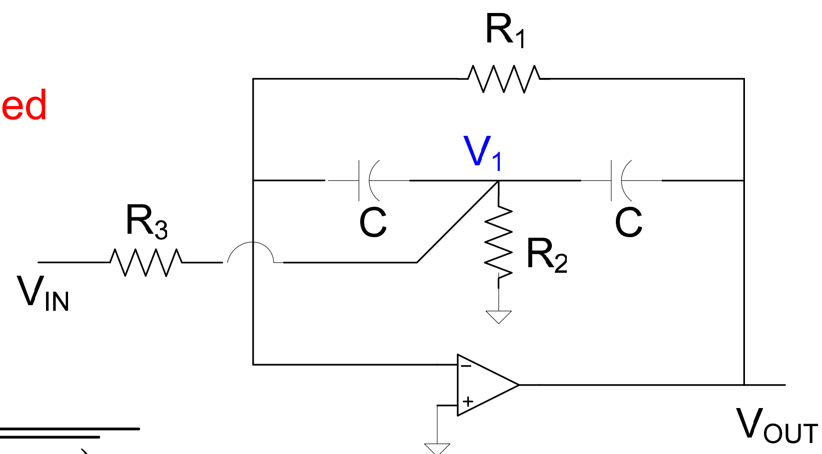


Another 2nd-order Bandpass Filter

Design Strategy

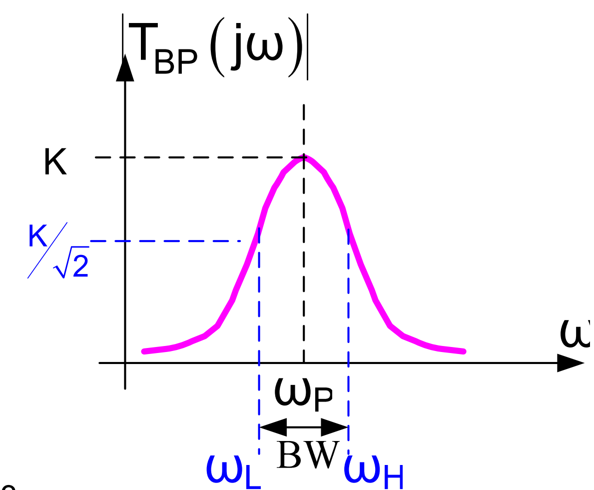
Assume BW, ω_P , and K are specified

$$T(s) = -\frac{\frac{s}{R_3C}}{s^2+s\left(\frac{2}{R_1C}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$



$$BW = \frac{2}{R_1C} \qquad \omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$$

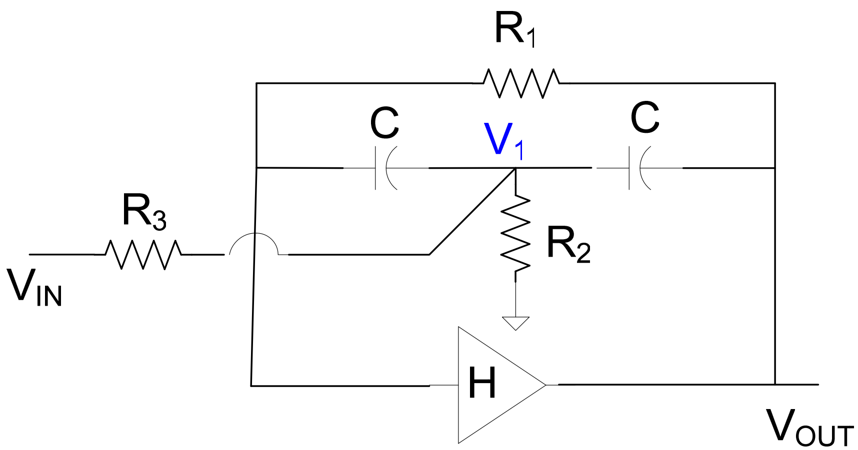
$$K = \frac{R_1}{2R_3}$$



1. Pick C to some practical or convenient value
2. Solve expression $BW = \frac{2}{R_1C}$ to obtain R_1
3. Solve expression $K = \frac{R_1}{2R_3}$ to obtain α and thus R_3
4. Solve expression $\omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$ to obtain R_2

Another 2nd-order Bandpass Filter

Termed the “STAR” biquad by inventors at Bell Labs



$$V_1\left(sC+sC+G_2+G_3\right)=V_{OUT}sC+V_{IN}G_3+V_{OUT}\frac{sC}{H}$$

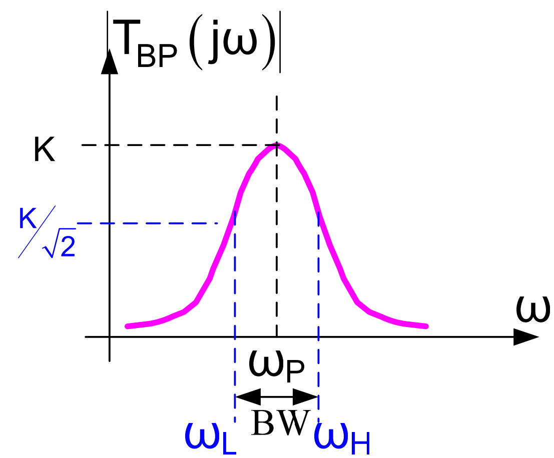
$$\frac{V_{OUT}}{H}\left(sC+G_1\right)=V_1sC+V_{OUT}G_1$$

$$T(s)=-\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

$$\omega_P=\frac{1}{\sqrt{R_1\left(R_2//R_3\right)}\,C}$$

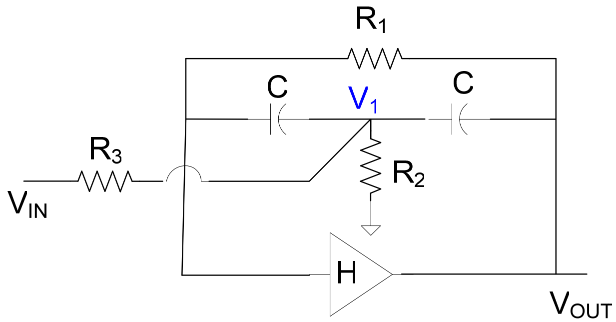
$$BW=\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}$$

$$K=\frac{\frac{1}{R_3}\left(\frac{H}{H-1}\right)}{\left(\frac{2}{R_1}-\frac{1}{(R_2//R_3)(H-1)}\right)}$$



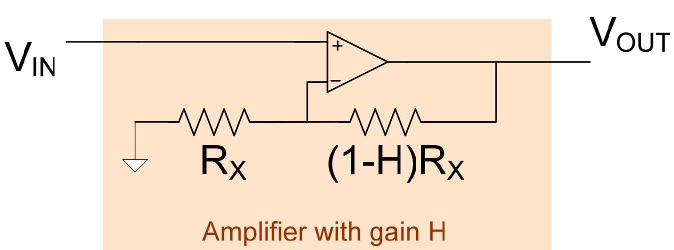
For the appropriate selection of component values, this is one of the best 2nd order bandpass filters that has been published

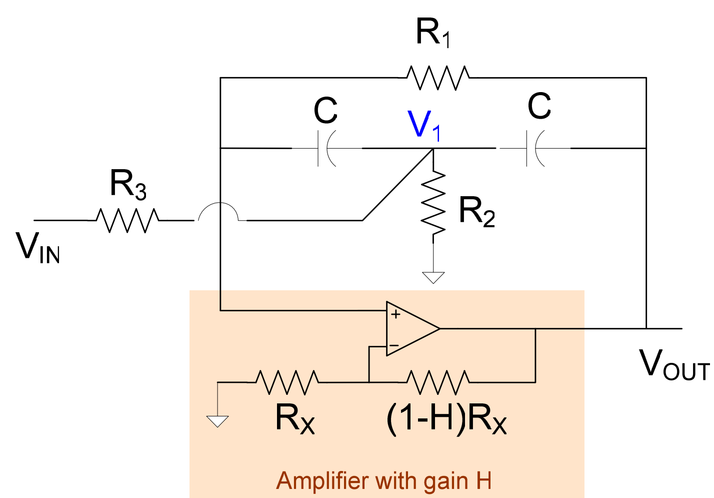
STAR 2nd-order Bandpass Filter



$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

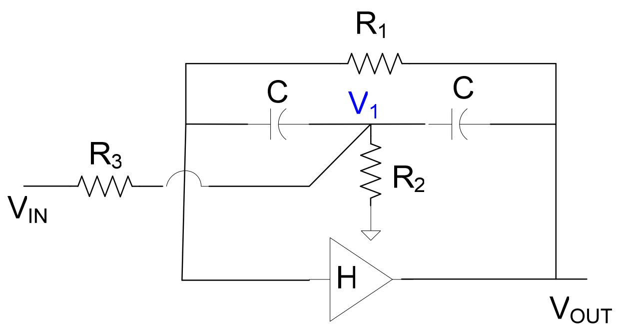
Implementation:





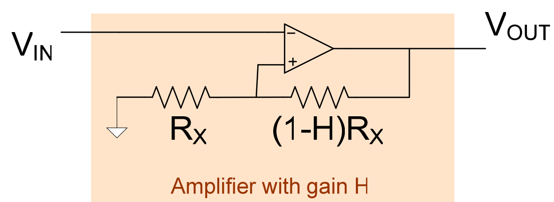
But the filter doesn't work !

STAR 2nd-order Bandpass Filter

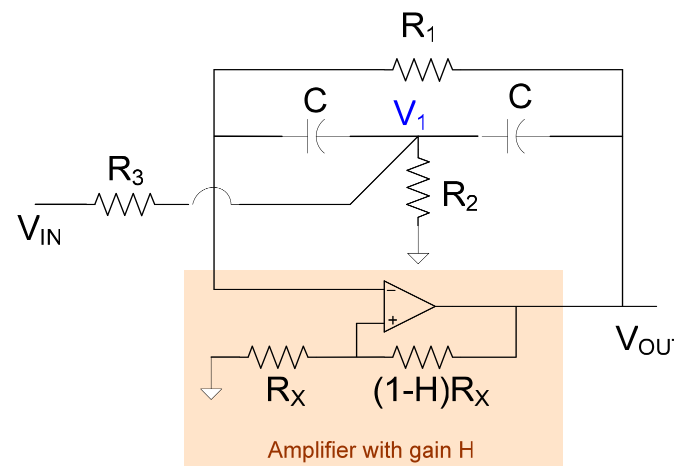


$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

Implementation:



If op amp ideal, $\frac{V_{OUT}}{V_{IN}} = H$



Works fine !

Will discuss why this happens later!

Reduces to previous bandpass filter at H gets large

Note that the “H” amplifier has feedback to positive terminal